Calculus III. Test 1 Review.

Also make sure to study your 3 quizzes!

1.

Given $\quad P = (5, 3, -1); \quad Q = (-2, 1, 0); \quad \vec{\mathbf{r}}(t) = \left\langle \sin(e^{3t} - 1),\ 7^{(2t)},\ \ln(e^t + 5) \right\rangle.$

(a) Find the parametric equations for the line that goes through Q and is perpendicular to the plane $7z - 4y = 11x + 21$.

(b) Find the plane through point P and perpendicular to the line $-x = 4y = \frac{z}{2}$. Simplify by collecting the constants on the right of your answer.

(c) Find the parametric equations for the tangent line to $\vec{\mathbf{r}}(t)$ at $t = 0$.

2. Given for a spaceship:

$\vec{\mathbf{r}}(3) = \langle 0, -11, -8 \rangle$, $\vec{\mathbf{r}}'(3) = \langle 1, 2, 1 \rangle$, $\vec{\mathbf{T}}'(3) = \langle 1, 0, -1 \rangle$, $a_T(3) = 5$.

(a) Find $\kappa(3)$.
(b) Find $\vec{\mathbf{T}}(3)$.
(c) Find $a_N(3)$.
(d) Find $\vec{\mathbf{N}}(3)$.
(e) Find the speed at $t = 3$.
(f) Is the spaceship speeding up or slowing down at $t = 3$?
(g) Find the acceleration at $t = 3$.

3. Given $\vec{\mathbf{r}}(t) = \langle t^2 + t,\ 5,\ -3t \rangle$.

   (a) Find the t-value of the max curvature.

(b) Find $a_T(1)$.
(c) Find $a_N(2)$.
(d) Find the velocity at $t = 3$
(e) Set up the integral for the arc length from $t = 0$ to $t = 5$.
(f) Is the spaceship speeding up or slowing down at $t = 1$?

4. Given for a spaceship located at $\vec{\mathbf{r}}(3) = \langle 9, 0, -4 \rangle$:

   $\vec{\mathbf{a}}(3) = \langle 0, -11, -8 \rangle$, $\vec{\mathbf{T}}(3) = \langle 0, 1, 0 \rangle$, $\vec{\mathbf{N}}(3) = \langle 0, 0, -1 \rangle$, and speed $= \frac{1}{4}$.

   (*a*) Find $a_T(3)$.
   (*b*) Find $\kappa(3)$.
   (*c*) Find $a_N(3)$.
   (*d*) Is the spaceship speeding up or slowing down at $t = 3$?
   (*e*) Find $\vec{\mathbf{v}}(3)$.
   (*f*) Find parametric equations for the tangent line at $t = 3$.

5. Given:

   $\vec{\mathbf{r}}(3) = \langle 11, -8, 0 \rangle$, $\vec{\mathbf{r}}'(3) = \langle 3, 0, -2 \rangle$, $\vec{\mathbf{N}}(3) = \langle 0, 1, 0 \rangle$, $a_N(3) = 2$, and $a_T(3) = -4$.

   (a) Find the tangent line to the curve $\vec{\mathbf{r}}(t)$ at $t = 3$. Give parametric equations for the line.

   (b) Find the acceleration $\vec{\mathbf{a}}(3)$.

   (c) Is the spaceship speeding up or slowing down at $t = 3$?