Version, Review

1.

Given $\quad P = (5, 3, -1); \quad Q = (-2, 1, 0); \quad \vec{\mathbf{r}}(t) = \left\langle \sin(e^{3t} - 1),\ 7^{(2t)},\ \ln(e^t + 5) \right\rangle.$

(*a*) Find the parametric equations for the line that goes through Q and is perpendicular to the plane $7z - 4y = 11x + 21$.

(*b*) Find the plane through point P and perpendicular to the line $-x = 4y = \frac{z}{2}$. Simplify by collecting the constants on the right of your answer.

a) $\vec{n} = \langle -11, -4, 7 \rangle$

$$\boxed{L = \begin{cases} x = -2 - 11t \\ y = 1 - 4t \\ z = 7t \end{cases}}$$

b) $\vec{n} = \langle -1, \frac{1}{4}, 2 \rangle$

$$\boxed{-1(x-5) + \frac{1}{4}(y-3) + 2(z+1) = 0}$$

$$-x + 5 + \frac{1}{4}y - \frac{3}{4} + 2z + 2 = 0$$

$$\boxed{-x + \frac{1}{4}y + 2z = \frac{-25}{4}}$$

(*c*) Find the parametric equations for the tangent line to $\vec{\mathbf{r}}(t)$ at $t = 0$.

$$\boxed{\vec{r}'(t) = \left\langle \cos(e^{3t} - 1)3e^{3t},\ 7^{2t}\ln 7(2),\ \frac{1}{e^t + 5}e^t \right\rangle}$$

$\vec{r}(0) = \langle 0, 1, \ln 6 \rangle$

$\vec{r}'(0) = \langle 3, 2\ln 7, \frac{1}{6} \rangle$

$$\boxed{L = \begin{cases} x = 3t \\ y = 1 + (2\ln 7)t \\ z = \ln 6 + \frac{1}{6}t \end{cases}}$$

Version Review

2. Given for a spaceship:

$\vec{\mathbf{r}}(3) = \langle 0, -11, -8\rangle$, $\vec{\mathbf{r}}'(3) = \langle 1, 2, 1\rangle$, $\vec{\mathbf{T}}'(3) = \langle 1, 0, -1\rangle$, $a_T(3) = 5$.

(*a*) Find $\kappa(3)$.
(*b*) Find $\vec{\mathbf{T}}(3)$.
(*c*) Find $a_N(3)$.
(*d*) Find $\vec{\mathbf{N}}(3)$.
(*e*) Find the speed at $t = 3$.
(*f*) Is the spaceship speeding up or slowing down at $t = 3$?
(*g*) Find the acceleration at $t = 3$.

a) $\kappa = \dfrac{|\vec{T}'|}{|\vec{r}'|} = \dfrac{\sqrt{2}}{\sqrt{6}} = \boxed{\sqrt{\frac{1}{3}}}$

b) $\vec{T} = \dfrac{\vec{r}'}{|\vec{r}'|} = \boxed{\frac{1}{\sqrt{6}}\langle 1, 2, 1\rangle}$

c) $a_N = |\vec{T}'||\vec{r}'| = \sqrt{2}\sqrt{6} = \boxed{\sqrt{12}}$

d) $\vec{N} = \dfrac{\vec{T}'}{|\vec{T}'|} = \boxed{\frac{1}{\sqrt{2}}\langle 1, 0, -1\rangle}$

e) $|\vec{r}'| = \boxed{\sqrt{6}}$

f) $\boxed{\text{speeding up}}$

g) $a = 5\dfrac{1}{\sqrt{6}}\langle 1,2,1\rangle + \sqrt{12}\,\dfrac{1}{\sqrt{2}}\langle 1,0,-1\rangle$

$= \boxed{\left\langle \frac{5}{\sqrt{6}} + \sqrt{6},\ \frac{10}{\sqrt{6}},\ \frac{5}{\sqrt{6}} - \sqrt{6} \right\rangle}$ $= \left\langle \frac{11}{\sqrt{6}}, \frac{10}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \right\rangle$

Version Review

3. Given $\vec{\mathbf{r}}(t) = \langle t^2 + t,\ 5,\ -3t \rangle$.

(a) Find the t-value of the max curvature.

$\vec{r}\,'(t) = \langle 2t+1, 0, -3 \rangle$ → $\vec{r}\,'(1) = \langle 3, 0, -3 \rangle$, $\vec{r}\,'(2) = \langle 5, 0, -3 \rangle$

$\vec{r}\,''(t) = \langle 2, 0, 0 \rangle$

$$\kappa = \boxed{\frac{6}{\sqrt{(2t+1)^2+9}^{\,3}} = 6(4t^2+4t+10)^{-3/2}}$$

$$\kappa' = \boxed{6\left(\frac{-3}{2}\right)(4t^2+4t+10)^{-5/2}(8t+4)} = 0$$

$\Rightarrow 8t+4 = 0$

$\boxed{t = \frac{-1}{2}}$

(b) Find $a_T(1)$.
(c) Find $a_N(2)$.
(d) Find the velocity at $t = 3$
(e) Set up the integral for the arc length from $t = 0$ to $t = 5$.
(f) Is the spaceship speeding up or slowing down at $t = 1$?

b) $a_T = \dfrac{\vec{r}\,'' \cdot \vec{r}\,'}{|\vec{r}\,'|} = \boxed{\dfrac{6}{\sqrt{18}}} = \sqrt{2}$

c) $a_N = \dfrac{|\vec{r}\,'' \times \vec{r}\,'|}{|\vec{r}\,'|} = |\langle 2,0,0 \rangle \times \langle 5,0,-3 \rangle| / |\vec{r}\,'| = \boxed{\dfrac{6}{\sqrt{34}}}$

d) $\vec{v}(3) = \vec{r}\,'(3) = \boxed{\langle 7, 0, -3 \rangle}$

e) $\boxed{\displaystyle\int_0^5 \sqrt{4t^2+4t+10}\, dt}$

f) $\boxed{\text{speeding up}}$

Version Review

4. Given for a spaceship located at $\vec{\mathbf{r}}(3) = \langle 9, 0, -4\rangle$:

$\vec{\mathbf{a}}(3) = \langle 0, -11, -8\rangle$, $\vec{\mathbf{T}}(3) = \langle 0, 1, 0\rangle$, $\vec{\mathbf{N}}(3) = \langle 0, 0, -1\rangle$, and speed $= \frac{1}{4}$.

(*a*) Find $a_T(3)$.
(*b*) Find $\kappa(3)$.
(*c*) Find $a_N(3)$.
(*d*) Is the spaceship speeding up or slowing down at $t = 3$?
(*e*) Find $\vec{\mathbf{v}}(3)$.
(*f*) Find parametric equations for the tangent line at $t = 3$.

a) $a_T = \vec{a} \cdot \vec{T} = \boxed{-11}$

b) $\kappa = \dfrac{a_N}{|\vec{r}\,'|^2} = \dfrac{8}{\left(\frac{1}{4}\right)^2} = 8 \cdot 16 = 80 + 48 = \boxed{128}$

c) $a_N = \vec{a} \cdot \vec{N} = \boxed{8}$

d) $\boxed{\text{slowing down}}$

e) $\vec{v} = \frac{1}{4}\langle 0, 1, 0\rangle = \boxed{\langle 0, \tfrac{1}{4}, 0\rangle}$

f) $\boxed{L = \begin{cases} x = 9 \\ y = t \\ z = -4 \end{cases}}$ OR $\boxed{L = \begin{cases} x = 9 \\ y = t/4 \\ z = -4 \end{cases}}$

Version Review

5. Given:

$\vec{r}(3) = \langle 11, -8, 0 \rangle$, $\vec{r}'(3) = \langle 3, 0, -2 \rangle$, $\vec{N}(3) = \langle 0, 1, 0 \rangle$, $a_N(3) = 2$, and $a_T(3) = -4$.

(a) Find the tangent line to the curve $\vec{r}(t)$ at $t = 3$. Give parametric equations for the line.

(b) Find the acceleration $\vec{a}(3)$.

(c) Is the spaceship speeding up or slowing down at $t = 3$?

a) $$\boxed{L = \begin{cases} x = 11 + 3t \\ y = -8 \\ z = -2t \end{cases}}$$

b) $$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

$$= -4 \frac{\langle 3, 0, -2 \rangle}{\sqrt{9+4}} + 2 \langle 0, 1, 0 \rangle$$

$$= \boxed{\left\langle \frac{-12}{\sqrt{13}}, 2, \frac{8}{\sqrt{13}} \right\rangle}$$

c) $\boxed{\text{slowing down.}}$