Calculus III. Test 1, Additional Review

1. For each function $z = f(x,y)$ find two partial derivatives: f_x and f_y.

(a) $f(x,y) = \dfrac{\ln y}{x}$

$f_x = \boxed{\dfrac{-\ln y}{x^2}}$ $f_y = \boxed{\dfrac{1}{xy}}$

(b) $f(x,y) = (y-2)^{(e^x)}$

$f_x = \boxed{(y-2)^{e^x}\ln(y-2)\,e^x}$ $f_y = \boxed{e^x(y-2)^{(e^x-1)}}$

(c) $f(x,y) = \sqrt[2y]{x-2}$ $= (x-2)^{\left(\frac{1}{2y}\right)}$

$f_x = \boxed{\dfrac{1}{2y}(x-2)^{\left(\frac{1}{2y}-1\right)}}$ $f_y = \boxed{(x-2)^{\frac{1}{2y}}\ln(x-2)\left(\dfrac{-1}{2y^2}\right)}$

(d) $f(x,y) = \sin(xy)$

$f_x = \boxed{y(\cos(xy))}$ $f_y = \boxed{x(\cos(xy))}$

(e) $f(x,y) = x^2 6^y$

$f_x = \boxed{2x6^y}$ $f_y = \boxed{x^2 6^y \ln 6}$

3. Given $z = f(x, y)$

Also $f(7,8) = 15$

and $f_x(7,8) = 5$

and $f_y(7,8) = -2$

a) Find the directional derivative of f over $(7,8)$ in the direction of $\vec{v} = \langle 3, -1 \rangle$.

$$D_{\vec{v}} f = \frac{\nabla f \cdot \vec{v}}{|\vec{v}|} = \frac{\langle 5,-2\rangle \cdot \langle 3,-1\rangle}{\sqrt{9+1}} = \boxed{\frac{17}{\sqrt{10}}}$$

b) Find the 2d direction vector of max decrease for $f(x, y)$ over $(x, y) = (7, 8)$.

$$-\nabla f = \boxed{\langle -5, 2\rangle}$$

c) Find the max rate of increase for f over $(7,8)$.

$$|\nabla f| = |\langle 5,-2\rangle| = \sqrt{25+4} = \boxed{\sqrt{29}}$$

e) For a curve $\vec{r}(t)$ obeying $\vec{r}(5) = \langle 7, 8\rangle$ and $\vec{r}'(5) = \langle 3, -7\rangle$; find $\frac{dz}{dt}$ when $t = 5$.

$$\frac{dz}{dt} = \nabla f \cdot \vec{r}' = \langle 5,-2\rangle \cdot \langle 3,-7\rangle = \boxed{29}$$

4. **Given $z = f(x, y)$ has a horizontal tangent plane over $(1, 2)$ and it has tangent plane given by $3x - 2y - z = 5$ over $(7, 8)$.**

$$z = 3x - 2y - 5 \Rightarrow z_0 = 3(7) - 2(8) - 5 = 0$$

a) Find the instantaneous rate of change in z at $(7, 8)$, as x increases and y is held constant at 8.

$$f_x(7,8) = \boxed{3}$$

since $\vec{n} = \langle \underset{f_x}{3}, \underset{f_y}{-2}, -1 \rangle$

b) Find the instantaneous rate of change in z with respect to t at $t = 0$ where (x, y) is on the curve $\vec{r}(t) = \langle t^2 + 7, 8e^t, \rangle$.

$$\vec{r}\,' = \langle 2t, 8e^t \rangle$$

$$\vec{r}\,'(0) = \langle 0, 8 \rangle$$

$$\frac{dz}{dt} = \nabla f \cdot \vec{r}\,'$$

$$= \langle 3, -2 \rangle \cdot \langle 0, 8 \rangle = \boxed{-16}$$

c) Use the linearization of $f(x, y)$ over $(7, 8)$ to approximate $f(7.1, 7.9)$.

$$L(7.1, 7.9) = f_x(7.1 - 7) + f_y(7.9 - 8) + z_0$$

$$= 3(0.1) + -2(-0.1) + 0 = \boxed{0.5}$$

$\boxed{\text{OR} \quad z = 3(7.1) - 2(7.9) - 5 = 0.5}$

e) Find the normal vector to the tangent plane of f at $(1, 2)$.

horizontal (given)

so ... $\boxed{\vec{n} = \langle 0, 0, -1 \rangle}$