Calculus III. Test 1, Additional Review

1. For each function $z = f(x, y)$ find two partial derivatives: f_x and f_y.

(a) $f(x, y) = \dfrac{\ln y}{x}$

$f_x =$ $f_y =$

(b) $f(x, y) = (y - 2)^{(e^x)}$

$f_x =$ $f_y =$

(c) $f(x, y) = \sqrt[2y]{x - 2}$

$f_x =$ $f_y =$

(d) $f(x, y) = \sin(xy)$

$f_x =$ $f_y =$

(e) $f(x, y) = x^2 6^y$

$f_x =$ $f_y =$

3. **Given** $z = f(x, y)$

Also $f(7, 8) = 15$

and $f_x(7, 8) = 5$

and $f_y(7, 8) = -2$

a) **Find the directional derivative of** f **over** $(7, 8)$ **in the direction of** $\vec{v} = \langle 3, -1 \rangle$.

b) **Find the 2d direction vector of max decrease for** $f(x, y)$ **over** $(x, y) = (7, 8)$.

c) **Find the max rate of increase for** f **over** $(7, 8)$.

e) **For a curve** $\vec{r}(t)$ **obeying** $\vec{r}(5) = \langle 7, 8 \rangle$ **and** $\vec{r}'(5) = \langle 3, -7 \rangle$; **find** $\frac{dz}{dt}$ **when** $t = 5$.

4. Given $z = f(x, y)$ has a horizontal tangent plane over $(1, 2)$ and it has tangent plane given by $3x - 2y - z = 5$ over $(7, 8)$.

a) Find the instantaneous rate of change in z at $(7, 8)$, as x increases and y is held constant at 8.

b) Find the instantaneous rate of change in z with respect to t at $t = 0$ where (x, y) is on the curve $\vec{r}(t) = \langle t^2 + 7, 8e^t, \rangle$.

c) Use the linearization of $f(x, y)$ over $(7, 8)$ to approximate $f(7.1, 7.9)$.

e) Find the normal vector to the tangent plane of f at $(1, 2)$.