Making a million with a little Calc 2 (and a second major/minor/5-year Master's in Math)!

Recall the p-series. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ **converges for p>1.**

What does the p-series converge to?

P=1	$\sum_{n=1}^{\infty} \frac{1}{n^1} =$	∞
P=2	$\sum_{n=1}^{\infty} \frac{1}{n^2} =$	$\frac{\pi^2}{6}$ These were discovered by Euler!
P=3	=	$1.2020569032\ldots$ Open question: Is this number transcendental? Apery's constant.
P=4	=	$\frac{\pi^4}{90}$
P=5	=	$1.0369277551\ldots$ Open question: Is this number irrational?
P=6	=	$\frac{\pi^6}{945}$
P=7	=	$1.0083492774\ldots$
P=8	=	$\frac{\pi^8}{9450}$
P=9	=	$1.0020083928\ldots$
P=10	=	$\frac{\pi^{10}}{93555}.$

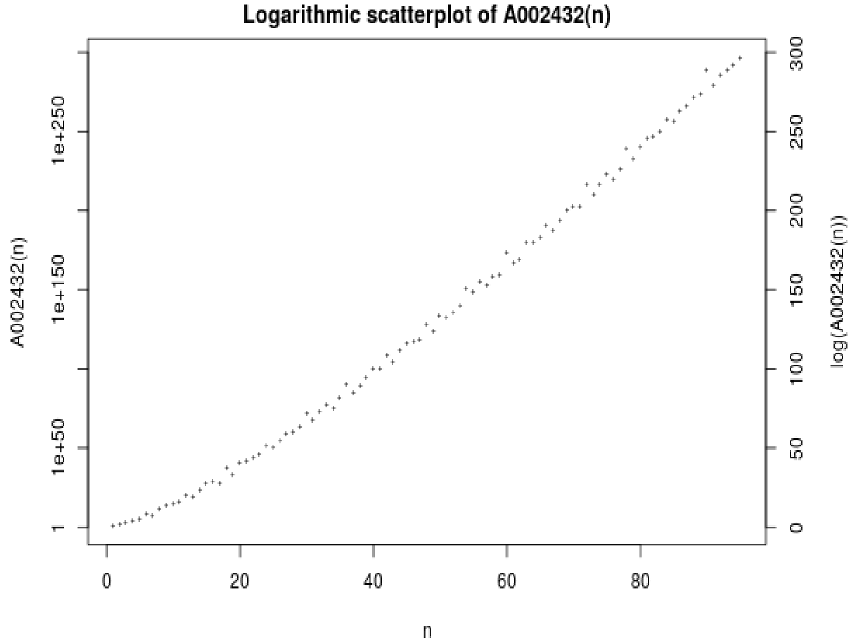
For more, see http://mathworld.wolfram.com/RiemannZetaFunction.html

Or… find about about that Master's degree:
http://www.uakron.edu/math/academics/undergraduate/index.dot#fund

Here are some more denominators for the values of p-series with even p.

(numerator = , $c\pi^{p}$ where c is found at http://oeis.org/A046988)

6,
90,
945,
9450,
93555,
638512875,
18243225, **Decrease**
325641566250,
38979295480125,
1531329465290625,
13447856940643125,
201919571963756521875,
11094481976030578125, **Decrease**
564653660170076273671875,
5660878804669082674070015625,
62490220571022341207266406250,
1213045458143374858729289062 5 **etc!**



For more info see http://oeis.org/A002432

There is a function called the Riemann zeta function ζ(p) which is equal to the p-series for p>1 but which can also give a value for any complex number input p=a+b*i* (except p=1.) It is the *analytic continuation* of the p-series. Here's a formula by H. Hasse:

$$\zeta(s) = \frac{1}{s-1}\sum_{n=0}^{\infty}\frac{1}{n+1}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+1)^{1-s}$$

(Here *p* is *s*. There's also a faster formula. Try some calculations, with real numbers: http://www.math.uakron.edu/~sf34/mandelbrot/riemann_fast.htm)

$\zeta(-2) = 0$
ζ(1/2 + 14.134725…*i*) = 0 Sometimes the answer is zero!

We can reinterpret 1+2+3+4+… as ζ(-1), if we agree that the divergent sum should be replaced by ζ. Turns out: ζ(-1) = -1/12 (This can be found several ways. However, any method requires "re-assigning" divergent series the values of certain analytic continuations. Cool thing though—sometimes nature seems to do that reassignment itself when virtual particles are involved…look up Hawking radiation and the Casimir effect.)

How to win $1,000,000!

Answer the following:
When does the analytic continuation of the p-series for complex numbers (the zeta function) equal 0?

It's already known that the answer includes all the negative even integers, and conjectured that the real part of any non-trivial zero of the Riemann zeta function is ½. If you want you can prove this equivalent theorem, the inequality:

$$\left|\pi(x) - \int_2^x \frac{dt}{\ln(t)}\right| \le C\sqrt{x}\,\ln(x)$$

where π(x) is the number of primes less than or equal to x, and *C* = 1/(8π) for all *x* ≥ 2657.

Lots more cool stuff is connected, including quasicrystals. More information on what is already known can be found at http://en.wikipedia.org/wiki/Riemann_hypothesis