| More Convergence testing | Requirements for application. | If this is true... | Then we conclude |
| --- | --- | --- | --- |
| Ratio test | Any, but useful with $n!$ and $x^n$. | $\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = L < 1$ | $\sum_{n=1}^{\infty} a_n$ converges absolutely. |
| | | $\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = L > 1$ | $\sum_{n=1}^{\infty} a_n$ diverges |
| | | $\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = 1$ | inconclusive |
| Root test | Any, but useful when there's an overall power of n. | $\lim_{n\to\infty} \|a_n\|^{\frac{1}{n}} = L < 1$ | $\sum_{n=1}^{\infty} a_n$ converges absolutely |
| | | $\lim_{n\to\infty} \|a_n\|^{\frac{1}{n}} = L > 1$ | $\sum_{n=1}^{\infty} a_n$ diverges |
| | | $\lim_{n\to\infty} \|a_n\|^{\frac{1}{n}} = 1$ | inconclusive |
| Ratio test for power series | looks like: $\sum_{n=0}^{\infty} c_n x^n$ or $\sum_{n=0}^{\infty} c_n (x-a)^n$ | Use Ratio test: same conclusion as above with $L < 1$, but $L$ is a function of $x$. Solve to find radius $R$ around $a$. | |
| End points for power series | Plug in $a+R$ and $a-R$ get $\sum c_n R^n$ and $\sum c_n (-R)^n$ | Use any of: alt. series test, lim test for divergence, geometric series, or p-series to decide which endpoint(s) converge / diverge. | |