

| Tests<br>11.2 - 11.6.*          | Requirements<br>for application  | If this is<br>true...   | Then we<br>conclude...   |
|---------------------------------|--|---|--|
| limit test<br>for<br>divergence | Any $a_n$  | $\lim_{n \rightarrow \infty} a_n \neq 0$<br>$\lim_{n \rightarrow \infty} a_n = 0$   | $\sum a_n$ diverges<br>inconclusive  |
| geo. series                     | $a_n = r^n$  | $ r  < 1$<br>$ r  \geq 1$   | $\sum_{n=1}^{\infty} (r)^n$ converges<br>to $\frac{r}{1-r}$<br>$\sum (r)^n$ diverges |
| p-series                        | $a_n = \frac{1}{n^p}$  | $p > 1$<br>$p \leq 1$   | $\sum a_n$ converges<br>$\sum a_n$ diverges  |
| integral<br>test                | $a_n = f(n)$ ; $f(x) > 0$ ,<br>continuous and<br>decreasing on $[1, \infty)$ | $\int_1^{\infty} f(x) dx$ converges<br>$\int_1^{\infty} f(x) dx$ diverges   | $\sum a_n$ converges<br>$\sum a_n$ diverges  |
| comparison<br>test              | $a_n > 0$<br>Known $b_n > 0$   | $a_n \leq b_n$ , $\sum b_n$ converges<br>$a_n \geq b_n$ , $\sum b_n$ diverges   | $\sum a_n$ converges<br>$\sum a_n$ diverges  |
| limit<br>comparison<br>test     | $a_n > 0$<br>Known $b_n > 0$   | $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ , $0 < L < \infty$<br>and $\sum b_n$ converges<br><br>$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ , $0 < L < \infty$<br>and $\sum b_n$ diverges<br><br>$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ or $\infty$ , or DNE | $\sum a_n$ converges<br><br>$\sum a_n$ diverges<br><br>inconclusive                  |
| alternating<br>series           | $a_n > 0$<br>$a_{n+1} \leq a_n$  | $\lim_{n \rightarrow \infty} a_n = 0$   | $\sum (-1)^n a_n$ converge   |
| *absolute<br>convergence        | Any $a_n$  | otherwise   | inconclusive   |
|                                 |  | $\sum_{n=1}^{\infty}  a_n $ converges<br>$\sum_{n=1}^{\infty}  a_n $ diverges   | $\sum a_n$ converges<br>inconclusive   |
| combinations                    | Any $a_n, b_n, c \in \mathbb{R}$ ,<br>$d \in \mathbb{R}$                     | $\sum a_n$ converges and<br>$\sum b_n$ converges  | $\sum (ca_n + db_n)$ converges   |