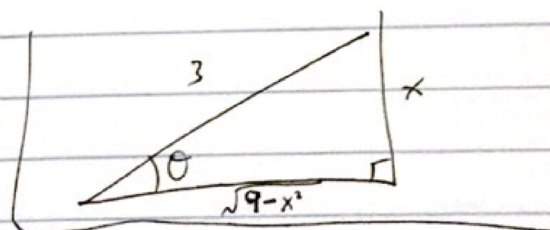
7.2 continued

Review $\int \text{trig} \, dx$

$\int \sin^4 x \, dx$	$\int \sin^4 x \cos^6 x \, dx$	$\int \sin^7 x \cos^4 x \, dx$	$\int \sin^8 x \cos^5 x \, dx$
$\to \int (\sin^2 x)^2 \, dx$	$\int (\sin^2 x)^2 (\cos^2 x)^3 \, dx$	$\int \sin^6 x \sin x \cos^4 x \, dx$	$\int \sin^8 x \cos^4 x \cos x \, dx$
$\to \sin^2 x = \frac{1-\cos 2x}{2}$	"	$\int (\sin^2 x)^3 \sin x \cos^4 x$	$\int \sin^8 x (\cos^2 x)^2 \cos x \, dx$
$\to \cos^2 x = \frac{1+\cos 2x}{2}$	"	$\to \sin^2 x = 1 - \cos^2 x$	$\to \cos^2 x = 1 - \sin^2 x$
		$u = \cos x$	$u = \sin x$
		$du = -\sin x \, dx$	$du = \cos x \, dx$

7.3 trig substitution.



(2) $$\int \frac{\sqrt{9-x^2}}{x^2} \, dx$$

let $x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$x^2 = 9\sin^2\theta$

$$= \int \frac{3\cos\theta}{9\sin^2\theta} 3\cos\theta \, d\theta$$

$$= \int \frac{\cos^2\theta}{\sin^2\theta} \, d\theta$$

$$= \int \cot^2\theta \, d\theta$$

$$= \int \csc^2\theta - 1 \, d\theta = -\cot\theta - \theta + c$$

$$= -\sqrt{9-x^2}/x - \sin^{-1}\left(\frac{x}{3}\right) + C$$

Or, for the numerator, use the triangle: find cosine as adjacent over hyp.

$$\sqrt{9-x^2} = \sqrt{9 - 9\sin^2\theta}$$
$$= \sqrt{9(1-\sin^2\theta)}$$
$$= 3\sqrt{\cos^2\theta}$$
$$= 3\cos\theta$$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$

(1) $\int \sqrt{1-x^2}\, dx$

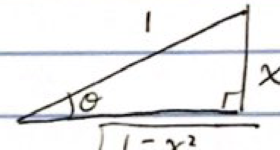
$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$1 - x^2 = 1 - \sin^2\theta = \cos^2\theta$

$= \int \sqrt{\cos^2\theta}\ \cos\theta\, d\theta$

$= \int \cos^2\theta\, d\theta$



$= \int \frac{1}{2} + \frac{\cos 2\theta}{2}\, d\theta$

$\theta = \sin^{-1} x$

$\cos\theta = \sqrt{1-x^2}$

$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \quad + C$

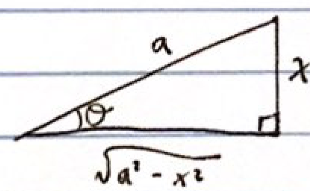
$= \frac{1}{2}\theta + \frac{1}{4} 2\sin\theta\cos\theta \quad + C$

$= \frac{1}{2}\sin^{-1} x + \frac{1}{2} x\sqrt{1-x^2} \quad + C$

check : derivative

MORE :

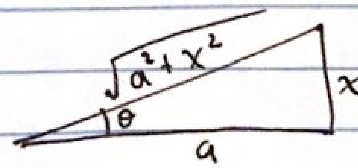
$\sqrt{a^2 - x^2}$ $\quad x = a\sin\theta$



Use

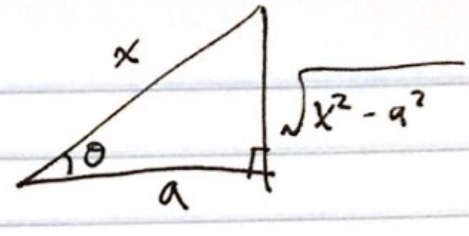
$1 - \sin^2 = \cos^2\theta$

$\sqrt{a^2 + x^2}$ $\quad x = a\tan\theta$



Use $1 + \tan^2\theta = \sec^2\theta$

$\sqrt{x^2 - a^2}$ $\quad$ $x = a\sec\theta$



Use $\sec^2\theta - 1 = \tan^2\theta$

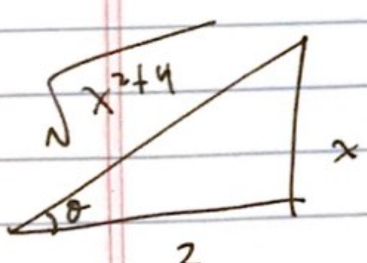
Ex: $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$ (ex 3, pg 480)

$$x = 2\tan\theta$$
$$dx = 2\sec^2\theta\,d\theta$$
$$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4}$$
$$= \sqrt{4\sec^2\theta}$$
$$= 2\sec\theta$$

$$= \int \frac{1}{4}\,\frac{\sec\theta}{\tan^2\theta}\,d\theta = \int \frac{1}{4}\,\frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$
$du = \cos\theta\,d\theta$

$$= -\frac{1}{4u} + C$$



$$= \frac{-1}{4\sin\theta} + C$$

$$= \frac{-1}{4\left(\frac{x}{\sqrt{x^2+4}}\right)} = \frac{-\sqrt{x^2+4}}{4x} + C$$