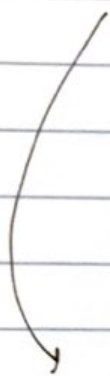


7.1 cont.

$$\int e^x \sin x \, dx$$



$$\left. \begin{array}{l} u = e^x \quad dv = \sin x \, dx \\ du = e^x \, dx \quad v = -\cos x \end{array} \right|$$

$$= -e^x \cos x - \int -\cos x e^x \, dx$$

$$= -e^x \cos x + \int \cos x e^x \, dx$$

$$\left. \begin{array}{l} u = e^x \quad dv = \cos x \, dx \\ du = e^x \, dx \quad v = \sin x \end{array} \right|$$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$\text{So } \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$
$$+ \int e^x \sin x \, dx \qquad + \int e^x \sin x \, dx$$

$$\Rightarrow 2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\Rightarrow \int e^x \sin x \, dx = \boxed{\frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + c}$$

$$\int_0^1 \tan^{-1} x \, dx$$

$$u = \tan^{-1} x$$

$$dv = dx$$

$$du = \frac{1}{1+x^2} dx$$

$$v = x$$

$$= \left[ x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx$$

$$- \int \frac{1}{2} \left( \frac{1}{u} \right) du \quad \left| \begin{array}{l} u = 1+x^2 \\ du = 2x \, dx \\ \frac{1}{2} du = x \, dx \end{array} \right.$$
$$- \left[ \frac{1}{2} \ln|u| \right]_{x=0}^{x=1}$$

$$= \left[ x \tan^{-1} x \right]_0^1 - \left[ \frac{1}{2} \ln|1+x^2| \right]_{x=0}^{x=1}$$

$$= \frac{\pi}{4} - 0 - \left( \frac{1}{2} \ln 2 - \frac{1}{2} \ln 1 \right)$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$

Review

$$\sin 2x = 2 \sin x \cos x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

7.2 cont.

$$\int \sin^5 x \cos^2 x \, dx$$

$$= \int (\sin^2 x)^2 \cos^2 x \sin x \, dx$$

$$= \int (1 - \cos^2 x)^2 \cos^2 x \sin x \, dx$$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$-du = \sin x \, dx$$

$$= \int (1 - u^2)^2 u^2 (-du)$$

$$= -\int (1 - 2u^2 + u^4) u^2 \, du$$

$$= -\int (u^2 - 2u^4 + u^6) \, du$$

$$= -\left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right) + C$$

$$= -\frac{1}{3} \cos^3 x + \frac{2}{5} \cos^5 x - \frac{1}{7} \cos^7 x + C$$