7.1 cont.

$$\int e^x \sin x \, dx$$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x \, dx \qquad v = -\cos x$

$$= -e^x \cos x - \int -\cos x \, e^x \, dx$$

$$= -e^x \cos x + \int \cos x \, e^x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x dx \qquad v = \sin x$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

So $\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$

$+ \int e^x \sin x \, dx \qquad\qquad + \int e^x \sin x \, dx$

$$\Rightarrow \quad 2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\Rightarrow \quad \int e^x \sin x \, dx = \boxed{-\frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C}$$

$$\int_0^1 \tan^{-1} x \, dx$$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$$= \left[x \tan^{-1} x\right]_0^1 - \underbrace{\int_0^1 \frac{x}{1+x^2} \, dx}$$

$$\underbrace{-\int \frac{1}{2}\left(\frac{1}{u}\right) du}$$

$u = 1+x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$$-\left[\frac{1}{2} \ln|u|\right]_{x=0}^{x=1}$$

$$= \left[x \tan^{-1} x\right]_0^1 - \left[\frac{1}{2} \ln\left|1+x^2\right|\right]_{x=0}^{x=1}$$

$$= \frac{\pi}{4} - 0 - \left(\frac{1}{2} \ln 2 - \frac{1}{2} \ln 1\right)$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$

Review

$$\sin 2x = 2 \sin x \cos x$$

$$\sin^2 x = \frac{1-\cos 2x}{2}, \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

7.2 cont.

$$\int \sin^5 x \cos^2 x \, dx$$

$$= \int (\sin^2 x)^2 \cos^2 x \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \cos^2 x \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= \int (1-u^2)^2 u^2 (-du)$$

$$= -\int (1-2u^2+u^4) u^2 \, du$$

$$= -\int (u^2 - 2u^4 + u^6) \, du$$

$$= -\left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right) + C$$

$$= -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$$