7.1 Integration by parts

"Reversing half the product rule."

ex:

$$y = x^2 e^{3x}$$

$$y' = 2x e^{3x} + x^2(3e^{3x})$$

So $\int \left(2x e^{3x} + x^2(3e^{3x})\right) dx = x^2 e^{3x} + c$

Also $\int \underbrace{x^2}_{u} \underbrace{3e^{3x}\, dx}_{dv} = \underbrace{x^2}_{u} \underbrace{e^{3x}}_{v} - \int 2x \underbrace{e^{3x}}_{v} dx$

$\underbrace{2x\,dx}_{du}$

$$\boxed{\int u\,dv = uv - \int v\,du.}$$

7.1 cont.

Find $\int x \cos 5x \, dx$.

Step 1) identify u and dv as two factors of original integral.
Idea: choose u that has simpler derivative.

Step 2)

$u = x$ $\quad$ $dv = \cos 5x \, dx$ (everything else!)

$\downarrow$ differentiate $\quad$ $\downarrow$ integrate

$du = 1 \, dx$ $\quad$ $v = \int \cos 5x \, dx$

$= \frac{1}{5} \sin 5x$

Step 3) Use formula $\underbrace{\int u \, dv}^{\text{original problem}} = \underbrace{uv - \int v \, du}_{\text{answer-in-progress}}$

$\int x \cos 5x \, dx$

4) Finish.

$= x \frac{1}{5} \sin 5x - \int \frac{1}{5} \sin 5x \, dx$

$= x \frac{1}{5} \sin 5x - \frac{1}{5}\left(\frac{-1}{5} \cos 5x\right)$

$$\boxed{= \frac{x}{5} \sin 5x + \frac{1}{25} \cos 5x} + C$$

Find

$$\int_1^2 \ln x \quad dx$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$$= \Big[x\ln x\Big]_1^2 - \int_1^2 x\,\frac{1}{x}\,dx$$

$$= \Big[x\ln x\Big]_1^2 - \int_1^2 1\,dx$$

$$= \Big[x\ln x\Big]_1^2 - \Big[x\Big]_1^2$$

$$= 2\ln 2 - 1\ln 1 - (2-1)$$

$$= \boxed{2\ln 2 - 1}$$

Find $\int x^2 e^x\,dx$

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

$$= x^2e^x - \int 2x\,e^x dx$$

$$= x^2 e^x - 2\int x e^x dx$$

but .... $\int x e^x dx$

$= xe^x - \int e^x dx$

$= xe^x - e^x$ ....

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$$= x^2e^x - 2\left(xe^x - e^x\right) + c$$

$$= \boxed{x^2e^x - 2xe^x + 2e^x + c}$$