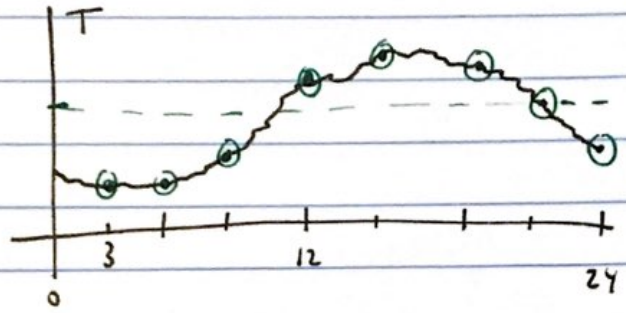
6.5 Average of a function

Example: average temperature in 24 hr period.



$$\text{average} = \frac{T(3) + T(6) + T(9) + T(12) + \cdots + T(21) + T(24)}{8}$$

since $\frac{24}{3} = 8 = n$, Δx

more data points $\dfrac{T(1) + T(2) + T(3) + \cdots + T(24)}{24}$

even more $\dfrac{T(.5) + T(1) + T(1.5) + \cdots + T(2)}{48}$

since $24/.5 = 48 = n$, Δx

in general $\dfrac{\sum_{i=1}^{n} T\left(0 + i\left(\frac{24-0}{n}\right)\right)}{(24-0)/\Delta x}$

$$= \frac{1}{24-0} \sum_{i=1}^{n} T\left(0 + i\left(\frac{24-0}{n}\right)\right) \Delta x$$

where $\Delta x = \frac{b-a}{n}$

for any function, and letting the number of data points go to ∞:

$$\text{average of } f(x) \text{ over } [a,b] = \left(\frac{1}{b-a}\right) \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + i\left(\frac{b-a}{n}\right)\right)$$

$$= \frac{1}{b-a} \int_a^b f(x)\, dx.$$

Ex: Find the average value of $\sqrt[3]{x}$ on the interval $[1,8]$

$$\text{Ave.} = \frac{1}{8-1} \int_1^8 \sqrt[3]{x}\, dx$$

$$= \frac{1}{7} \int_1^8 x^{1/3}\, dx$$

$$= \frac{1}{7} \left[\frac{3}{4} x^{4/3}\right]_1^8$$

$$= \frac{1}{7} \cdot \frac{3}{4} \left(8^{4/3} - 1^{4/3}\right)$$

$$= \frac{3}{28} \left(\left(\sqrt[3]{8}\right)^4 - 1\right)$$

$$= \frac{3}{28} (16 - 1)$$

$$= \boxed{\frac{45}{28}}$$