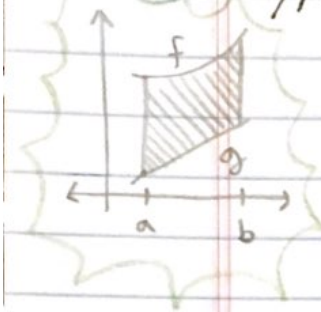
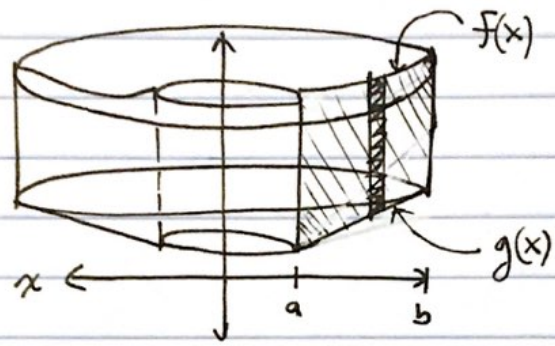
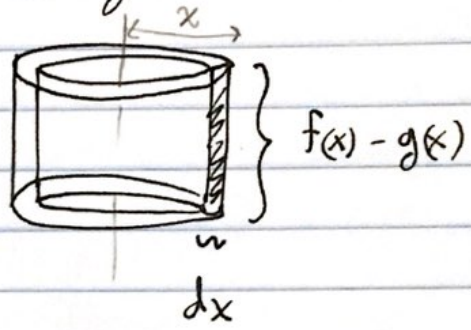
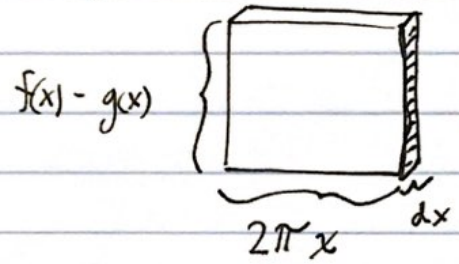
6.2 cont + 6.3

Type I region rotated around y-axis.





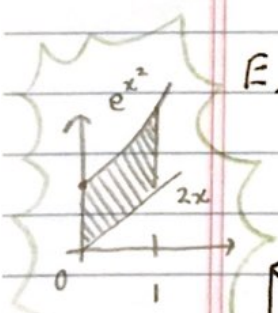


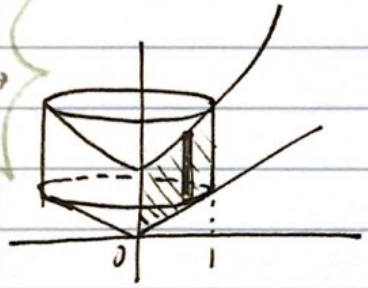


$$dV = 2\pi x\left(f(x) - g(x)\right)dx$$

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right)dx.$$

Ex: Find volume of region rotated around y axis. $\begin{cases} x=0\,,\ x=1 \\ y=2x\,,\ y=e^{x^2} \end{cases}$





$$dV = 2\pi x\left(e^{x^2} - 2x\right)dx$$

$$V = \int_0^1 2\pi x\left(e^{x^2} - 2x\right)dx$$

$$= 2\pi\int_0^1 \left(xe^{x^2} - 2x^2\right)dx$$

$$= 2\pi\left(\int_0^1 xe^{x^2}dx - \int_0^1 2x^2\,dx\right)$$

$$\begin{cases} u = x^2 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{cases}$$

$$= 2\pi\left(\int_{x=0}^{x=1} \frac{1}{2}e^u\,du - \int_0^1 2x^2\,dx\right)$$

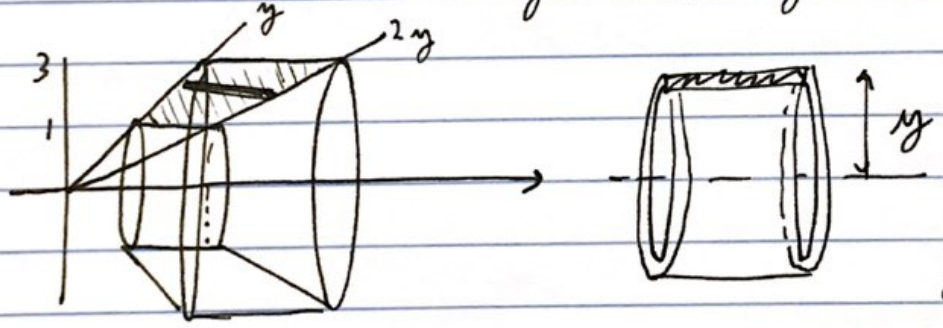
$$= 2\pi\left(\left[\frac{1}{2}e^{x^2}\right]_0^1 - \left[\frac{2x^3}{3}\right]_0^1\right)$$

$$= 2\pi\left(\frac{e}{2} - \frac{1}{2} - \frac{2}{3} - 0\right) = \pi(e-1) - \frac{4\pi}{3}$$

$$= \pi\left(e - \frac{7}{3}\right)$$

Type II region rotated around x-axis

Ex: region $\begin{cases} y = 1 & y = 3 \\ x = y & x = 2y \end{cases}$



$$dV = 2\pi y\left(f(y) - g(y)\right)dy$$

$$V = \int_1^3 2\pi y\left(2y - y\right)dy$$

$$= 2\pi\int_1^3 y^2\,dy = 2\pi\left[\frac{y^3}{3}\right]_1^3$$

$$= 2\pi\left(\frac{27}{3} - \frac{1}{3}\right) = 2\pi\left(\frac{26}{3}\right)$$