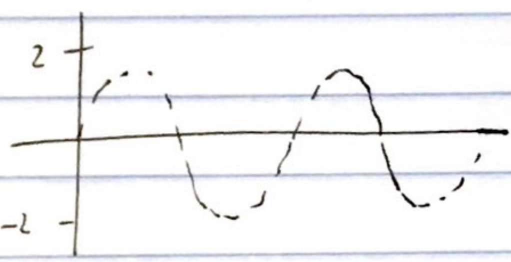
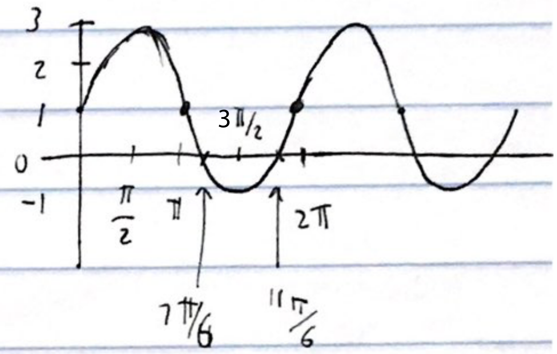
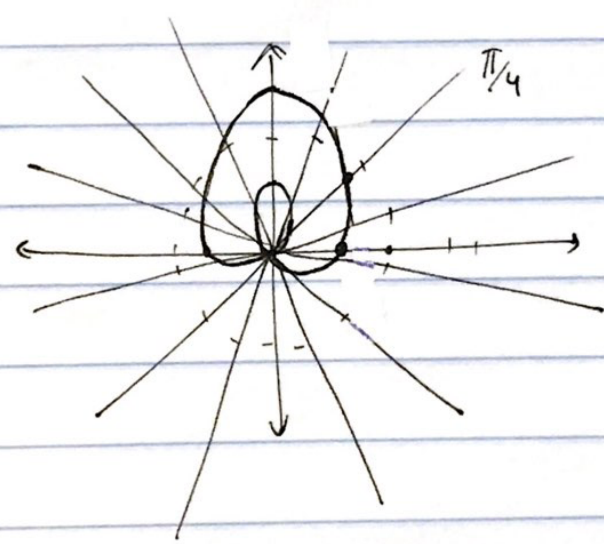
10.4 Areas + lengths in polar

Ex: $r = 1 + 2\sin\theta$







$$L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\ d\theta$$

simplifies to $\displaystyle\int_a^b \sqrt{(r(\theta))^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$

length of the limacon set up:

$$\int_0^{2\pi} \sqrt{(1+2\sin\theta)^2 + (2\cos\theta)^2}\ d\theta$$

Area inside a polar curve

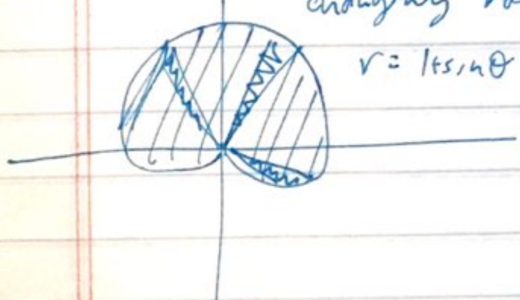
Circle Area = πr^2

Any slice of angle θ (sector)

$$\text{Area} = \pi r^2\left(\frac{\theta}{2\pi}\right) = \frac{1}{2}r^2\theta$$

changing radius:

$r = 1+\sin\theta$

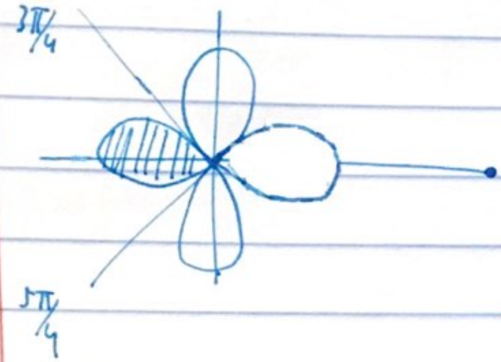


$$A = \int_0^{2\pi} \frac{1}{2}(r^2)\,d\theta$$

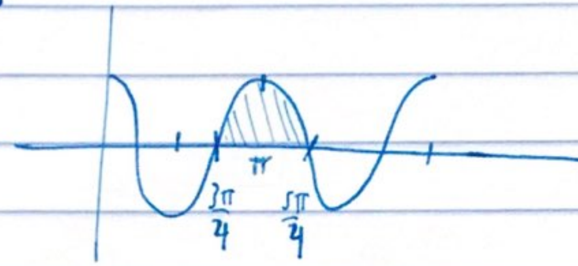
$$= \int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2\,d\theta$$

$$= \frac{3\pi}{2}$$

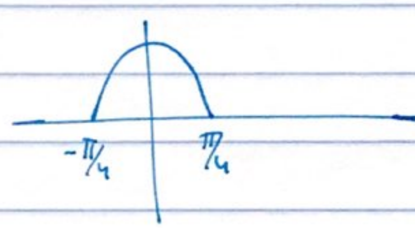
Area in one loop of the 4-leaved rose



$r = \cos 2\theta$



or alternately



$$\int_{3\pi/4}^{5\pi/4} \frac{1}{2} r^2 \, d\theta = \int_{\frac{3\pi}{4}}^{5\pi/4} \frac{1}{2}\left(\cos^2(2\theta)\right) d\theta$$

$$= \int \frac{1}{2}\left(\frac{1 + \cos 4\theta}{2}\right) d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{1}{4}\sin 4\theta\right]_{3\pi/4}^{5\pi/4}$$

$$= \frac{1}{4}\left(\frac{\pi}{2}\right) = \frac{\pi}{8}$$