Ch 10.2

Calculus with parametric curves

$C = \begin{cases} x = t^2 \\ y = t^3 - 3t \end{cases}$

Find $\frac{dy}{dx}$ at $t = 2$

Formula: $y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$y'' = \frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{dy'/dt}{dx/dt}$

$dy/dt = 3t^2 - 3$

$dx/dt = 2t$

$y' = \frac{dy}{dx} = \frac{3t^2-3}{2t}$

$\frac{dy'}{dt} = \frac{2t(6t) - 2(3t^2-3)}{4t^2}$

$= \frac{12t^2 - 6t^2 + 6}{4t^2}$

$= (6t^2+6)/4t^2$

$y'' = ((6t^2+6)/4t^2)/2t$

$= (3t^2+3)/4t^3$

at $t = 2 = \frac{3 \cdot 4 - 3}{4} = \frac{9}{4}$

Find max, min, crit. points:

$\frac{dy}{dx} = 0 \implies \frac{3t^2-3}{2t} = 0 \Rightarrow 3t^2 = 3$

$t^2 = 1$

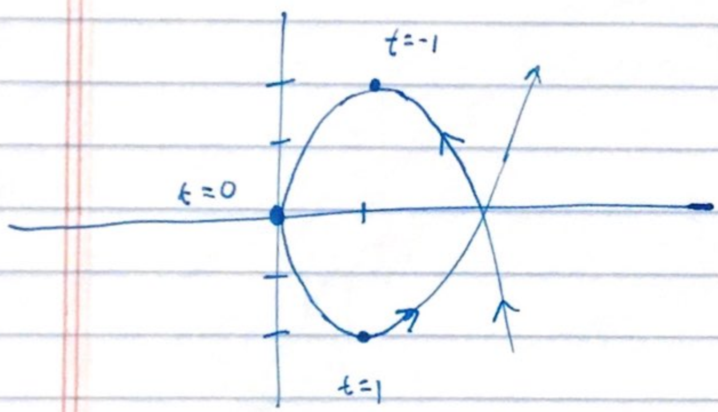
$\boxed{t = \pm 1}$

$\frac{dy}{dx} = DNE \Rightarrow 2t = 0 \Rightarrow \boxed{t = 0.}$ $\left(\frac{dy}{dx} = \pm\infty\right)$

t	-1	0	1
$\frac{d^2y}{dx^2} = \frac{3(t^2+1)}{4t^3}$	<0 max *	DNE	>0 min *
$(x, y) =$	$(1, 2)$	$(0,0)$	$(1, -2)$

* Second derivative test.

rough sketch



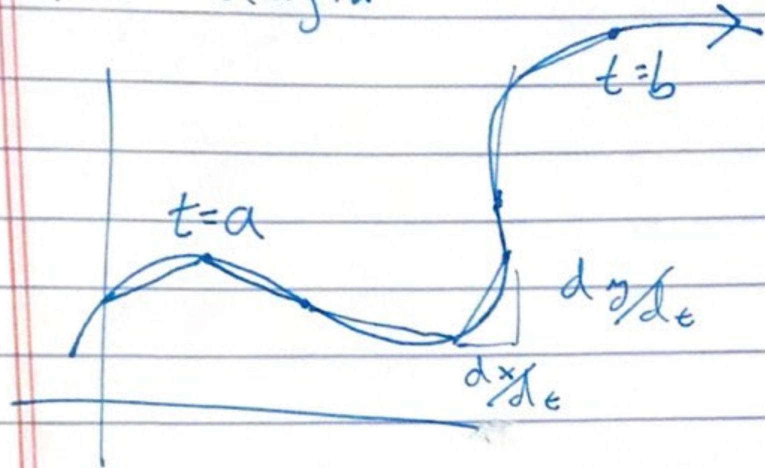
Crossing point? set $y = 0$

$t^3 - 3t = 0$

$t(t^2-3) = 0$

$t=0, \quad t = \pm\sqrt{3}$

Arc Length



$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

↑
arc length from $t=a$ to $t=b$.

Set up the arc length for

$$C = \begin{cases} x = e^t \\ y = \sin 7t \end{cases}, \quad t \in [-2, 5]$$

$$L = \int_{-2}^{5} \sqrt{(e^t)^2 + (7\cos 7t)^2}\, dt.$$

Ex. Find arc length of $y = e^{3x} + x^2$ from $x=1$ to $x=5$.
(set up)

1) Parameterize: Let $C = \begin{cases} x = t \\ y = e^{3t} + t^2 \end{cases}$ $\quad t \in [1,5]$

2)

$$L = \int_1^5 \sqrt{1 + (3e^{3t} + 2t)^2}\, dt$$