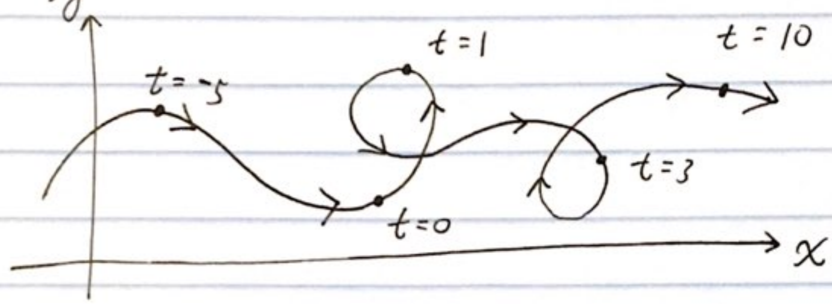
Chp. 10

parametric equations for curves



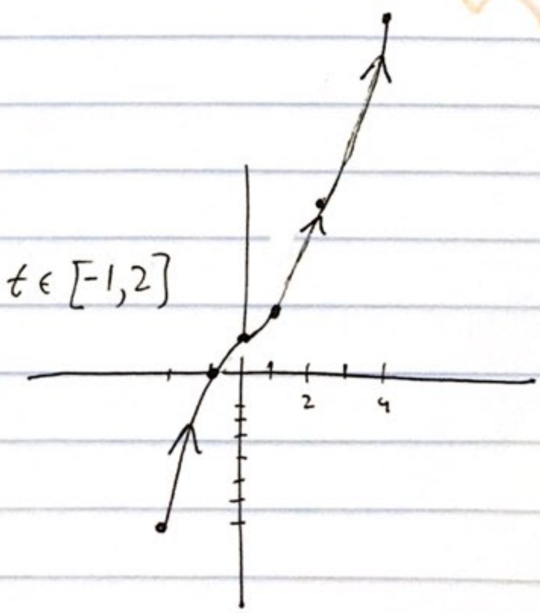
We can describe any curve
by telling where x and y are
for input t.

curve $C = \begin{cases} x = f(t) \\ y = g(t) \end{cases}, \ t \in [a,b]$ so position at $t = t_0$
is $(x, y) = (f(t_0), g(t_0))$

Sometimes we use $\begin{matrix} x = x(t) \\ y = y(t) \end{matrix}$ See Related Rates, 3.9

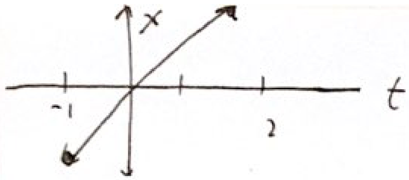
Ex:

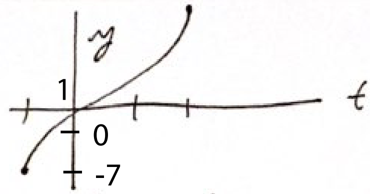
$$C = \begin{cases} x = 2t \\ y = 8t^3 + 1 \end{cases}, \ t \in [-1, 2]$$



t	x	y
$\frac{1}{2}$	1	2
$-\frac{1}{2}$	-1	0

t	x	y
0	0	1
-1	-2	-7
1	2	9
2	4	65



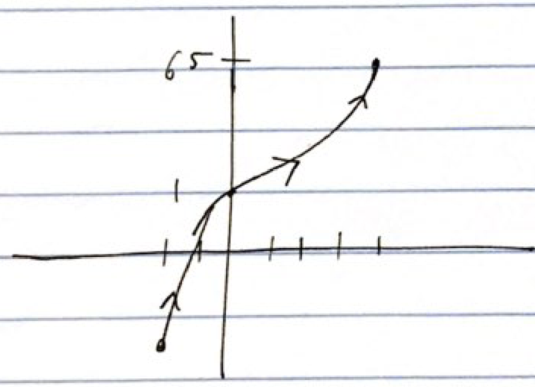


We can eliminate t to see the shape of the curve. This will give a clear picture of the path but not how fast and in what direction it is traced.

$$x = 2t \qquad y = 8\left(\frac{x}{2}\right)^3 + 1$$

$$\frac{x}{2} = t \qquad y = x^3 + 1$$

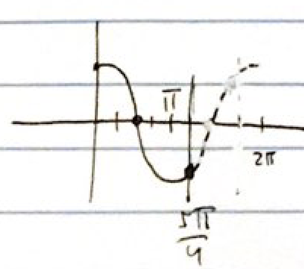
Putting together the shape and the plotted points / plotted x vs t and y vs t.

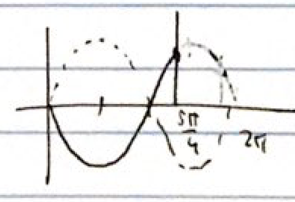


Ex:

$$\begin{cases} x = \cos t \\ y = -\sin t \end{cases} \qquad t \in \left[0, \frac{5\pi}{4}\right]$$

t	x	y
0	1	0
$\pi/4$	$\sqrt{2}/2$	$-\frac{\sqrt{2}}{2}$
$\pi/2$	0	-1
π	-1	0
$3\pi/2$	0	1
$5\pi/4$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

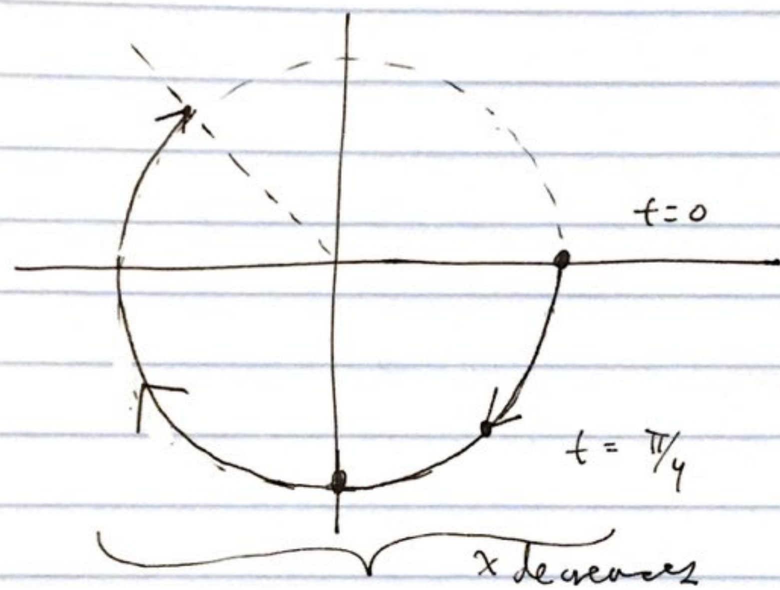




$x^2 = \cos^2 t$

$y^2 = \sin^2 t$

$x^2 + y^2 = 1$ circle radius 1



Ex $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$ $t \in [0, 2\pi]$