Ex: What power series converge to $f(x)$?

$f(x) = \dfrac{3x^2}{1+x}$? (in standard form)

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

sub

$$\Rightarrow \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n$$

$$\cdot 3x^2 \Rightarrow \frac{3x^2}{1+x} = \sum_{n=0}^{\infty} 3x^2(-x)^n$$

$$= \sum_{n=0}^{\infty} 3(-1)^n x^{n+2} = 3(-1)^0 x^2 + \cdots$$

$$= \boxed{\sum_{n=2}^{\infty} 3(-1)^{n-2} x^n} = 3(-1)^0 x^2 + \ldots$$

check

Ex: What power series does converge to

$\ln(1+x)$?

sub $-x$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-x)^n$$

$\int$

$$\ln(1+x) = \sum_{n=0}^{\infty} \int (-x)^n = \sum_{n=0}^{\infty} -\frac{(-x)^{n+1}}{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+2} x^{n+1}}{n+1}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

More convergent power series ...

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

demo : take derivatives of both: get e^x

$$\frac{d}{dx} \quad \sum_{n=0}^{\infty} \frac{n x^{n-1}}{n!} = \sum_{n=1}^{\infty} \frac{n x^{(n-1)}}{n(n-1)!} = \sum_{n=1} \frac{x^{n-1}}{(n-1)!}$$

replace n with n+1 : n+1 = 1 ⇒ n=0

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$ same as start!

For any function $f(x)$ which has all its derivatives defined :

Theorem

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!}}$$ [Maclaurin]

$$= f(0)\frac{1}{1} + f'(0)\frac{x}{1} + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{3!} + \ldots$$

for $e^x = f(x)$,

$f'(x) = e^x \quad f'(0) = 1$

$f''(x) = e^x \quad f''(0) = 1$

$f'''(x) = e^x$