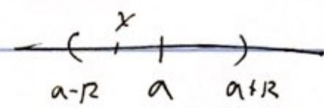
11.8 Power series:

series with a variable x
always found as a factor x^n
or $(x-a)^n$

$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

3 possibilities:
1) converges when $x = a$ only
2) converges for all x
3) converges for a radius R around a, so for all x in $(a-R, a+R)$

[Note: $(a-R, a+R)$ = x such that $|x-a| < R$



]

AND we need to check the endpoints to find the interval of convergence

possible answers:
$(a-R, a+R)$
$(a-R, a+R]$
$[a-R, a+R)$
$[a-R, a+R]$

Use: the ratio test.

Ex: Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n} \quad . \quad \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{n+1} \cdot \frac{n}{(x-3)^n} \right|$$

$$= \lim_{n\to\infty} \left| (x-3) \frac{n}{n+1} \right| = |x-3|$$

so $|x-3| < 1$

so for $x \in (3-1, 3+1) \Rightarrow$ convergent

Check endpoints $x = 2$, $x = 4$

$x=2$: $\sum_{n=1}^{\infty} \frac{(2-3)^n}{n} = \sum \frac{(-1)^n}{n}$ converges by alt. series

$x=4$ $\sum_{n=1}^{\infty} \frac{(4-3)^n}{n} = \sum \frac{1^n}{n} = \sum \frac{1}{n}$ diverges by p-series.

So answer is $[2, 4)$ or $2 \leq x < 4$.

$R = 1$

Ex: $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n\to\infty} \frac{|x|}{n+1} = 0 < 1$ for all x

so interval of convergence is $(-\infty, \infty)$, converges for all x.

$R = \infty$

Ex: $\sum_{n=1}^{\infty} 5n x^n$ $\lim_{n\to\infty} \left| \frac{5(n+1) x^{n+1}}{5n x^n} \right| = \lim_{n\to\infty} \frac{5n+5}{5n} |x|$

$= |x|$

So converges for $|x| < 1$. $(-1, 1)$

diverges at both ends by lim test for divergence.

$R = 1$

Ex:

$$\sum 3^n x^n$$

$$\lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{3^n x^n} \right| = \lim_{n\to\infty} |3x| = |3x|$$

$$|3x| < 1$$

$$|x| < \frac{1}{3}$$

$-1/3 \quad 0 \quad 1/3$ $\qquad R = 1/3$

diverges at both ends
by lim test for div.

Ex: $\sum 5n!\, x^n$

$$\lim_{n\to\infty} \left| \frac{5(n+1)!\, x^{n+1}}{5n!\, x^n} \right| = \lim_{n\to\infty} |(n+1)x|$$

$$= \begin{cases} \infty & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

so $R = 0$, interval of conv. $= [0,0]$.