Comparison test:

For $a_n > 0$, $b_n > 0$

(all output terms positive)

if $a_n \geq b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges

$\uparrow$ given seq. $\uparrow$ well known

then $\sum_{n=1}^{\infty} a_n$ diverges too.

if $a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges

then $\sum_{n=1}^{\infty} a_n$ converges too.

Ex: $a_n = \frac{7}{3n^2+2n}$ Does $\sum_{n=1}^{\infty} \frac{7}{3n^2+2n}$ converge?

Compare to $\frac{1}{n^2}$

Since $3n^2+2n > n^2$, $\frac{1}{3n^2+2n} < \frac{1}{n^2}$

and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series (to $\frac{\pi^2}{6}$)

So, $\sum_{n=1}^{\infty} \frac{7}{3n^2+2n} = 7 \sum_{n=1}^{\infty} \frac{1}{3n^2+2n}$

$= 7$ (real number).

It converges, by comparison to $\sum \frac{1}{n^2}$.

Ex: Does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} - \frac{1}{2}}$ converge?

Compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$, which diverges since $\frac{1}{2} < 1$.

since $\sqrt{n} - \frac{1}{2} < \sqrt{n}$, $\frac{1}{\sqrt{n} - \frac{1}{2}} > \frac{1}{\sqrt{n}}$

So, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} - \frac{1}{2}}$ diverges by comp. to $\sum \frac{1}{\sqrt{n}}$

Ex: Does $\sum_{n=1}^{\infty} \frac{3}{7(2^n) - 13}$ converge?

Note $7(2^n) - 13 \not> 2^n$ [although we could force this by discarding early terms]

so we introduce the <u>limit comparison test</u>

For $a_n > 0$, $b_n > 0$

if $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ (real number)

then either both diverge or both converge.

$$\lim_{n\to\infty} \frac{\frac{3}{7(2^n)-13}}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{3(2^n)}{7(2^n)-13} = \lim_{x\to\infty} \frac{3(2^x)\ln 2}{7(2^x)\ln 2} = \boxed{\frac{3}{7}}$$

(L'Hopitals)

so converge