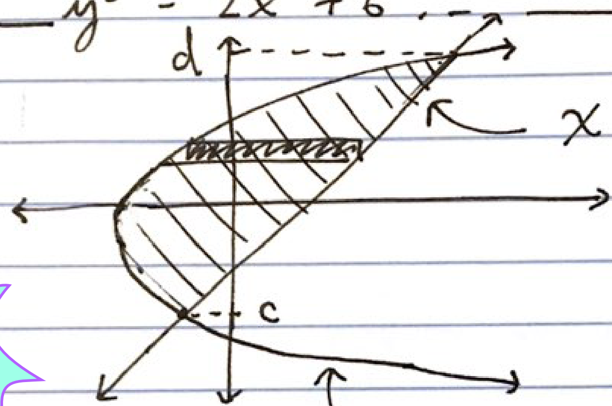
6.1 cont.

Example:

Find area between $y = x-1$ and $y^2 = 2x+6$.

Step 0)



Draw and label

$x = y+1$

$\frac{1}{2}y^2 = x+3$

$x = \frac{1}{2}y^2 - 3$

Step 1) Type II $\underbrace{\boxed{}}_{(y+1)-\left(\frac{1}{2}y^2-3\right)}\; \} dy$

Decide

Step 2) $A = \int_c^d (f(y) - g(y))\,dy$

Find c, d: $y+1 = \frac{1}{2}y^2 - 3$

Find values

II -> y-values

$y = \frac{1}{2}y^2 - 4$

$2y = y^2 - 8$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4$, $y = -2$

d c

Set-Up

Step 3: $A = \int_{-2}^{4}\left((y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right) dy$

Step 4:

Simplify and solve

$$= \int_{-2}^{4} \left(y + 4 - \tfrac{1}{2}y^2\right) dy$$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$$

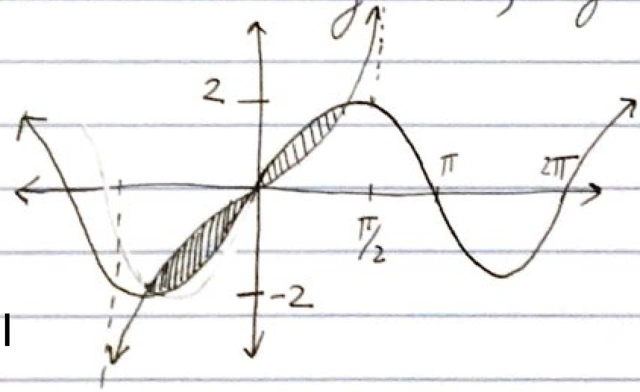
$$= \left(8 + 16 - \frac{64}{6}\right) - \left(\frac{4}{2} - 8 + \frac{8}{6}\right)$$

$$= 32 - 2 - 12 = \boxed{18}$$

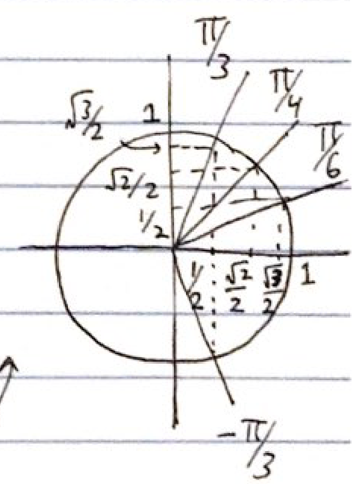
Example: Area between two curves that cross between the limits of integration.

Find area between $y = \tan x$, $y = 2\sin x$, $-\pi/3 \le x \le \pi/3$

0)



1) Type I



Intersections: $\tan x = 2\sin x$

2) $\Rightarrow \dfrac{\sin x}{\cos x} = 2\sin x$

$\sin x = 0$ | $\cos x = \frac{1}{2}$

$x = 0$ | $x = \pi/3, -\pi/3$

Now, between $-\pi/3$ and 0, $\tan x > 2\sin x$

but between 0 and $\pi/3$, $2\sin x > \tan x$.

Thus

3) $$A = \int_{-\pi/3}^{0} (\tan x - 2\sin x)\,dx + \int_{0}^{\pi/3} (2\sin x - \tan x)\,dx$$

Step 4 : solve

Recall $\int \tan x \, dx = \int \frac{\sin x}{\cos x}\,dx$ | $u = \cos x$, $du = -\sin x\,dx$

$$= \int \frac{-1}{u}\,du$$

$$= -\ln|u| + c$$

$$= -\ln|(\cos x)| + c$$

$-\ln a = \ln a^{-1}$

so $$= \ln|\sec x| + c$$

$$A = \Big[\ln|\sec x| + 2\cos x\Big]_{-\pi/3}^{0} + \Big[-2\cos x - \ln|\sec x|\Big]_{0}^{\pi/3}$$

$$= (\ln 1 + 2) - (\ln 2 + 1) + (-1 - \ln 2) - (-2 - \ln 1)$$

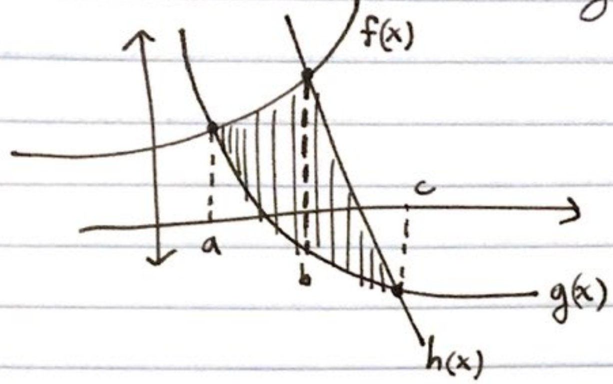
$$= 1 - \ln 2 \quad + \quad 1 - \ln 2$$

* Note: could have used symmetry (or double check)

$$= 2(1 - \ln 2)$$

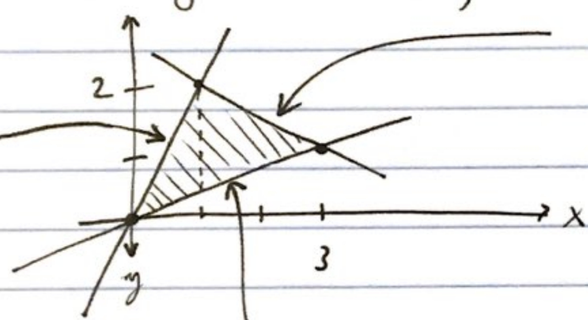
$$= 2 - 2\ln 2.$$

3 curves & subdividing



$$A = \int_a^b \left(f(x) - g(x)\right) dx + \int_b^c \left(h(x) - g(x)\right) dx$$

Ex: triangle $(0,0)$, $(3,1)$, $(1,2)$



$y = 2x$

$y = \frac{1}{3}x$

slope $= \frac{2-1}{1-3} = -\frac{1}{2}$

$y - 2 = -\frac{1}{2}(x-1)$

$y = -\frac{1}{2}x + \frac{1}{2} + 2$

$y = -\frac{1}{2}x + \frac{5}{2}$

$$A = \int_0^1 \left(2x - \left(\tfrac{1}{3}x\right)\right) dx + \int_1^3 \left(-\tfrac{1}{2}x + \tfrac{5}{2} - \tfrac{1}{3}x\right) dx$$

$$= \int_0^1 \frac{5}{3}x \, dx + \int_1^3 \left(-\frac{5}{6}x + \frac{5}{2}\right) dx$$

$$= \left[\frac{5}{6}x^2\right]_0^1 + \left[\frac{-5}{12}x^2 + \frac{5}{2}x\right]_1^3$$

$$= \frac{5}{6} - 0 + \left(\frac{-45}{12} + \frac{15}{2}\right) - \left(\frac{-5}{12} + \frac{5}{2}\right) = \boxed{\frac{5}{2}}$$