11.1 Sequences

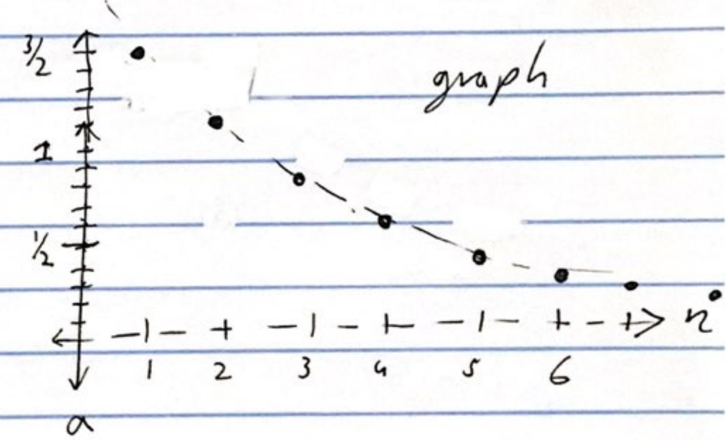
A <u>sequence</u> is a function from the whole numbers (natural numbers) to the real numbers.

Inputs: 1, 2, 3, 4, 5, ... (sometimes 0 too)

Outputs: any fraction, negatives, irrationals

Ex: $a = f(n) = \dfrac{3n}{1+n^2}$

n	$f(n) = a_n$
1	$\frac{3}{2}$
2	$\frac{6}{5}$
3	$\frac{9}{10}$
4	$\frac{12}{17}$
5	$\frac{15}{26}$



Note: $a_n > 0$ for all n.

Ex: $a_n = \sqrt{2n} - 3n$

n	a_n
1	$\sqrt{2} - 3$
2	$2 - 6 = -4$
3	$\sqrt{6} - 9$
4	$\sqrt{8} - 12$

limits:

We can take $\lim_{n\to\infty} a_n$.

We say $\lim_{n\to\infty} a_n = L$ when

there is a real number L that a_n gets closer and closer to as n gets larger.

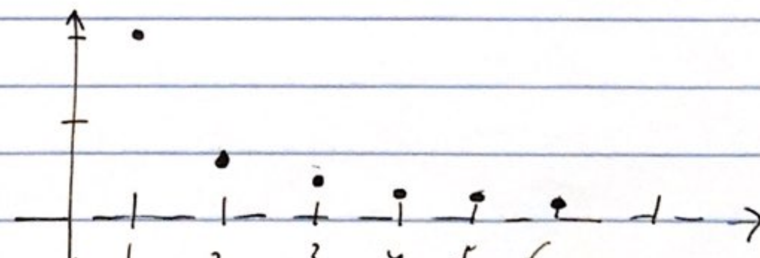
I.e. $\lim_{n\to\infty} a_n = L$ means:

for any $\varepsilon > 0$, there exists a natural number N such that

if $n > N$ then $|a_n - L| < \varepsilon$.

Ex: $a_n = \frac{1}{n^2}$

n	a_n
1	1
2	1/4
3	1/9
4	1/16
5	1/25
6	1/36



$\lim_{n\to\infty} a_n = 0$

Ex: for $\varepsilon = 0.1$, we can find $N = 4$, since $1/16 < 0.1$ we see $\left|\frac{1}{n^2} - 0\right| < 0.1$ for $n > 4$.

Ex:

$$a_n = e^n + 5$$

n	a_n
1	$e+5$
2	e^2+5
3	e^3+5
4	e^4+5
5	e^5+5

$$\lim_{n\to\infty} a_n = \infty$$

If $\lim_{n\to\infty} a_n = \pm\infty$ or DNE, we say a_n diverges.

If $\lim_{n\to\infty} a_n = L$ we say a_n converges.

Ex: $a_n = (-1)^n$

n	a_n
1	-1
2	1
3	-1
4	1
5	-1
6	1

$\lim_{n\to\infty} a_n =$ DNE since it gets closer to no unique L.

(divergent).