Improper integrals: two types of integrals involving ∞.

1) $x \to \infty$ (see example $\int_1^\infty \frac{1}{x^2}$)

Another example

$$\int_3^\infty \frac{1}{x}\,dx$$

$$= \lim_{t \to \infty} \int_3^t \frac{1}{x}\,dx$$

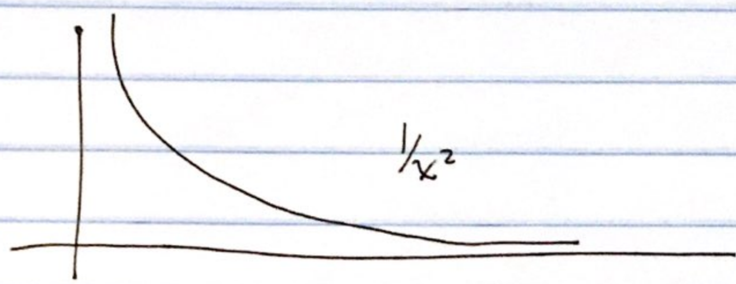
$$= \lim_{t \to \infty} \Big[\ln x\Big]_3^t$$

$$= \lim_{t \to \infty} (\ln t - \ln 3)$$

$$= \infty - \ln 3 = \infty$$

we say that the integral diverges if (is divergent) the answer is ∞.

7.8



$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx$$

$$= \lim_{t\to\infty} \left[-\frac{1}{x} \right]_1^t$$

$$= \lim_{t\to\infty} \left(-\frac{1}{t} - \frac{-1}{1} \right)$$

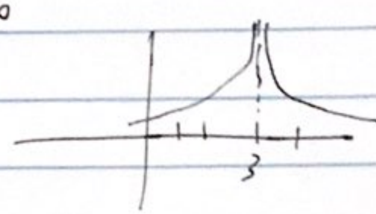
$$= \lim_{t\to\infty} \left(\frac{-1}{t} + 1 \right)$$

$$= 0 + 1$$

$$= 1 .$$

2) vertical asymptotes : $y \to \infty$

$$\int_1^4 \frac{1}{(x-3)^2}\,dx$$



Must be split up!!

$$= \underbrace{\int_1^3 \frac{1}{(x-3)^2}\,dx}\ + \int_3^4 \frac{1}{(x-3)^2}\,dx$$

$$= \lim_{t \to 3^-} \int_1^t \frac{1}{(x-3)^2}\,dx$$

$u = x-3$
$du = dx$

$$\lim_{t \to 3^-} \left[\frac{-1}{x-3}\right]_1^t$$

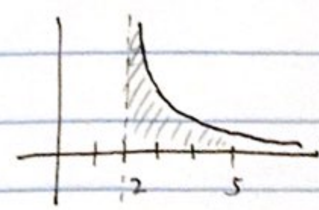
$$\lim_{t \to 3^-} \left(\frac{-1}{t-3} - \left(\frac{-1}{-2}\right)\right)$$

$$= \infty - \frac{1}{2}$$

$$= \infty$$

Ex:

$$\int_2^5 \frac{1}{\sqrt{x-2}}\,dx = 2\sqrt{3}$$



$\boxed{\begin{array}{l} u = x-2 \\ du = dx \end{array}}$

$$= \int_{x=2}^{x=5} \frac{1}{u^{1/2}}\,du = \int_{x=2}^{x=5} u^{-1/2}\,du = 2u^{1/2} = \left[2\sqrt{x-2}\right]_2^5$$

$$= 2\sqrt{3}$$

example

$$\int_0^3 \frac{1}{\sqrt{x}}\,dx = \lim_{t\to 0^+} \int_t^3 \frac{1}{\sqrt{x}}\,dx$$

$$= \lim_{t\to 0^+} \int_t^3 x^{-1/2}\,dx$$

$$= \lim_{t\to 0^+} \Big[2x^{1/2}\Big]_t^3$$

$$= \lim_{t\to 0^+} \left(2\sqrt{3} - 2\sqrt{t}\right)$$

$$= \boxed{2\sqrt{3}}$$