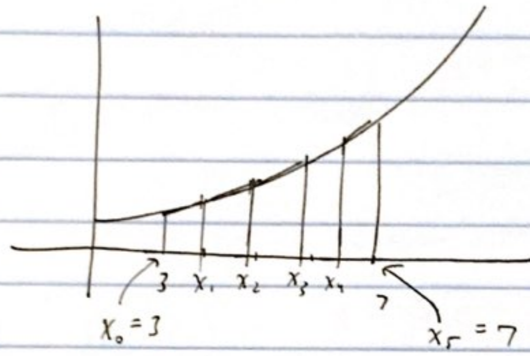
Ex. trapezoid rule.

$$\int_3^7 e^{x^2}\,dx \quad \text{from } 3 \text{ to } 7 \text{ with } 5 \text{ trapezoids}$$



$$\Delta x = \frac{7-3}{5} = \frac{4}{5}$$

$$\int_3^7 e^{x^2}\,dx \approx \sum_{i=1}^{5} \frac{\Delta x}{2}\left(f(x_{i-1}) + f(x_i)\right)$$

$$= \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_4) + f(x_5)\right)$$

$+\,4/5$

$$= \frac{7-3}{5}\left(\frac{1}{2}\right)\left(e^{3^2} + 2e^{\left(\frac{19}{5}\right)^2} + 2e^{\left(\frac{23}{5}\right)^2} + 2e^{\left(\frac{27}{5}\right)^2} + 2e^{\left(\frac{31}{5}\right)^2} + e^{\left(\frac{35}{5}\right)^2}\right)$$

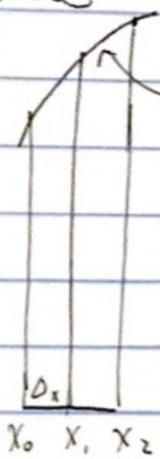
$$= \frac{4}{10}\left(e^{9} + 2e^{14.44} + 2e^{21.16} + 2e^{29.16} + 2e^{38.44} + e^{49}\right)$$

$$\boxed{= 7.6297820\,30382 \times 10^{20}}$$

Simpson's Rule

n = even

number of rectangles



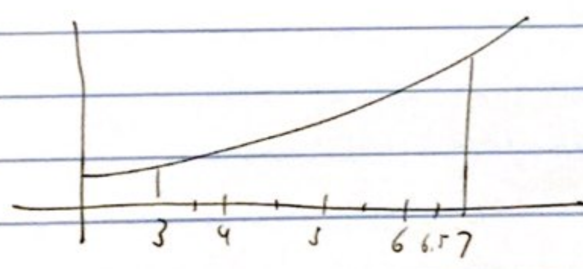
use a parabola piece to make the top, no matter what the original $f(x)$ is.

Two rectangles at once!

Area under parabola is $\frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right)$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big)$$

$$\int_3^7 e^{x^2}\,dx \qquad n = 8 \qquad \Delta x = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$$



$$\frac{1/2}{3}\left(e^{3^2} + 4e^{3.5^2} + 2e^{4^2} + 4e^{4.5^2} + 2e^{5^2} + 4e^{5.5^2} + 2e^{6^2} + 4e^{6.5^2} + e^{7^2}\right)$$

$= 3.1938 \times 10^{20}$