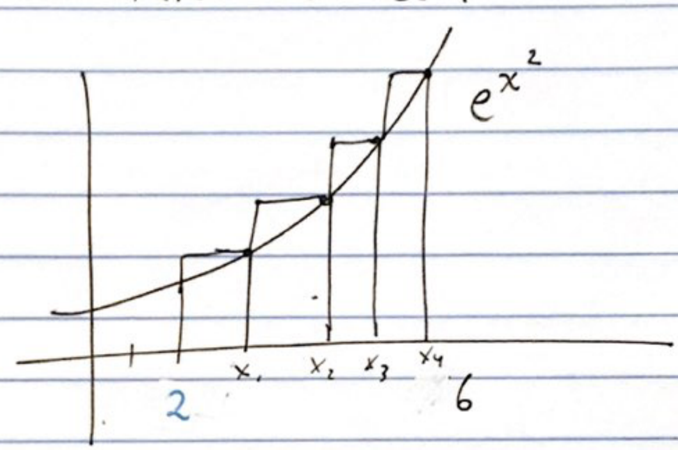
7.7 Approximate integration

Review : Riemann sum



$$x_i = a + i\left(\frac{b-a}{n}\right)$$

$$\int_2^6 e^{x^2}\,dx \approx \sum_{i=1}^{4} \underbrace{\left(\frac{6-2}{4}\right)}_{\substack{\text{base} \\ \Delta x = \frac{b-a}{n}}} e^{\left(2 + i\left(\frac{6-2}{4}\right)\right)^2}$$

number of rectangles n

height $= f(x_i)$

$$\left[\int_2^6 e^{x^2}\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6-2}{n}\right) e^{\left(2 + i\left(\frac{6-2}{n}\right)\right)^2}\right]$$

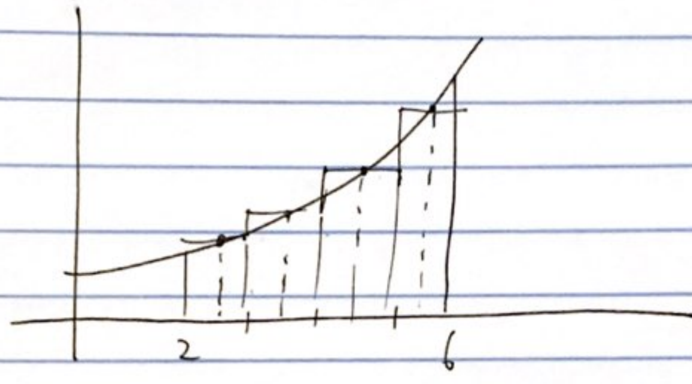
$$\approx \frac{4}{4} e^{(2+1)^2} + \frac{4}{4} e^{(2+2)^2} + 1\, e^{(2+3)^2} + 1\, e^{(2+4)^2}$$

$$= e^{9} + e^{16} + e^{25} + e^{36}$$

$$= 4{,}311{,}303{,}560{,}908{,}750.0 \quad (4 \text{ quadrillion})$$

Midpoint:

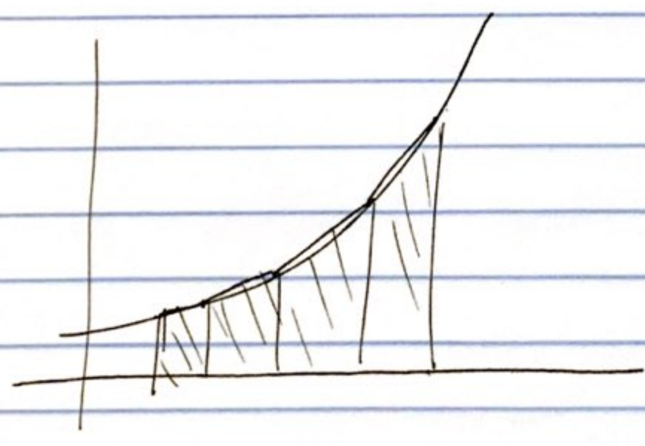
Instead use $\bar{x}_i = \frac{1}{2}\left(x_{i-1} + x_i\right) = a + i\left(\frac{b-a}{n}\right) - \frac{1}{2}\left(\frac{b-a}{n}\right)$

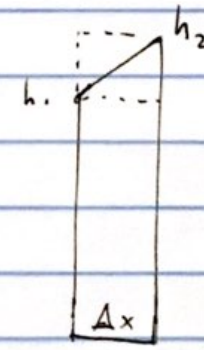


Left hand

Instead use $x_{(i-1)} = a + (i-1)\left(\frac{b-a}{n}\right)$

Trapezoid





$$\text{area} = \frac{\Delta x h_1 + \Delta x h_2}{2}$$

$$= \Delta x\left(\frac{h_1 + h_2}{2}\right) = \frac{\Delta x}{2}\left(h_1 + h_2\right)$$

$$= \frac{\Delta x}{2}\left(f(x_{i-1}) + f(x_i)\right)$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{\Delta x}{2}\left(\sum_{i=1}^{n}\left(f(x_{(i-1)}) + f(x_i)\right)\right)$$

$$= \frac{\Delta x}{2}\left(f(x_0) + \underbrace{f(x_1) + f(x_1)} + \underbrace{f(x_2) + f(x_2)} + \dots + f(x_n)\right)$$

$$\approx \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right)$$

ex: e^{x^2}