7.4 cont., 7.5

Recall $\int \frac{1}{x^2+9}\,dx = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

$$\int \frac{2x^2-x+4}{x^3+4x}\,dx$$

$$\boxed{\frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}}$$

$$\Rightarrow \quad 2x^2-x+4 = A(x^2+4) + (Bx+C)x$$

$$= Ax^2 + 4A + Bx^2 + Cx$$

$$= (A+B)x^2 + Cx + 4A$$

$\Rightarrow$ $A+B=2$

$4A = 4 \longrightarrow A=1$, sub: $1+B=2$

$C=-1$ $\qquad B=1$

$$\text{so} \quad \int \frac{2x^2-x+4}{x^3+4x}\,dx = \int \left(\frac{1}{x} + \frac{x-1}{x^2+4}\right)dx$$

$$= \int \frac{1}{x}\,dx + \int \frac{x-1}{x^2+4}\,dx$$

$$= \int \frac{1}{x}\,dx + \int \frac{x}{x^2+4}\,dx + \int \frac{-1}{x^2+4}\,dx$$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \ln|x| + \frac{1}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

Ex: $\boxed{\int \frac{x^3+2}{x^2-1}}$

$$\begin{array}{r l}
 & \quad x \qquad\qquad R\ \ x+2 \\
x^2-1 & \big)\ x^3+2 \\
 & -(x^3-x\qquad) \\
 & \qquad x+2
\end{array}$$

So $\boxed{\frac{x^3+2}{x^2-1} = x + \frac{x+2}{x^2-1}}$

and $\frac{x+2}{x^2-1} = \frac{x+2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$$x+2 = A(x+1) + B(x-1)$$
$$x+2 = Ax + A + Bx - B$$
$$= (A+B)x + A - B$$

$$A+B=1$$
$$A-B=2$$

$$2A = 3$$
$$A = 3/2, \quad B = 1 - \frac{3}{2} = -\frac{1}{2}$$

$$= \int x + \frac{3/2}{x-1} - \frac{1/2}{x+1}\,dx$$

$$= \frac{x^2}{2} + \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$