7.4 Partial Fractions

"Finding easier denominators"

"Reversing the process of finding a common denominator."

Recall: Ex. 1.

$$\frac{3}{x+5} + \frac{2}{2x-1} = \frac{3(2x-1)}{(x+5)(2x-1)} + \frac{2(x+5)}{(x+5)(2x-1)}$$

$$= \frac{3(2x-1) + 2(x+5)}{(x+5)(2x-1)}$$

$$= \frac{6x - 3 + 2x + 10}{2x^2 + 10x - x - 5}$$

$$= \frac{8x + 7}{2x^2 + 9x - 5}$$

OR

$$\frac{1}{x} + \frac{3}{x^2} + \frac{5}{x-1} = \frac{1 \cdot x(x-1)}{x^2(x-1)} + \frac{3(x-1)}{x^2(x-1)} + \frac{5x^2}{x^2(x-1)}$$

$$= \frac{x^2 - x + 3x - 3 + 5x^2}{x^2(x-1)}$$

$$= \frac{6x^2 + 2x - 3}{x^3 - x^2}$$

Now reverse it! Why?

Find: $$\int \frac{8x+7}{2x^2+9x-5}\,dx$$

1) factor denominator:

$$\frac{8x+7}{2x^2+9x-5} = \frac{8x+7}{(2x-1)(x+5)}$$

2) Since the factors are both linear (no squares) we can assume it came from fractions with constants on top:

$$\frac{8x+7}{(2x-1)(x+5)} = \frac{A}{2x-1} + \frac{B}{x+5}$$

3) Recall how we add, by getting common denominators:

$$\frac{8x+7}{(2x-1)(x+5)} = \frac{A(x+5)}{(2x-1)(x+5)} + \frac{B(2x-1)}{(2x-1)(x+5)}$$

4) Set the numerators equal:

$$8x+7 = A(x+5) + B(2x-1)$$

$$8x+7 = Ax + 5A + 2Bx - B$$

$$8x+7 = (A+2B)x + 5A - B$$

5) Now since this must be true for all x, we know :

$$7 = 5A - B$$

and so

$$8 = A + 2B$$

6) Solve! (substitute, or add equations to eliminate a variable)

$$\begin{aligned} 14 &= 10A - 2B \\ + \quad 8 &= A + 2B \\ \hline 22 &= 11A \\ \boxed{2 = A} & \end{aligned}$$

substitute

$$\begin{aligned} 8 &= 2 + 2B \\ 6 &= 2B \\ \boxed{3 = B} & \end{aligned}$$

Now we know:

$$\frac{8x+7}{2x^2+9x-5} = \frac{3}{x+5} + \frac{2}{2x-1}$$

(easy to check: redo example 1)

so

$$\int \frac{8x+7}{2x^2+9x-5}\,dx = \int\left(\frac{3}{x+5} + \frac{2}{2x-1}\right)dx$$

$$= 3\ln|x+5| + \frac{2\ln|2x-1|}{2} + c$$

$$= 3\ln|x+5| + \ln|2x-1| + C$$

In general: for every power of a factor of denominator, guess a numerator with 1 less degree.

Ex: Just find the form for partial fractions

① $$\frac{x^3+x^2+1}{x(x-1)(x^2+3x+5)(x^2+7)^3}$$

$$\to \frac{A}{x}+\frac{B}{x-1}+\frac{Cx+D}{x^2+3x+5}+\frac{Ex+F}{x^2+7}+\frac{Gx+H}{(x^2+7)^2}+\frac{Ix+J}{(x^2+7)^3}$$

② $$\frac{3x^2+5x+1}{(x^2+2x)(x^2-4)(x+1)^2}=\frac{3x^2+5x+1}{x\underline{(x+2)(x+2)}(x-2)(x+1)^2}$$

$$=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{x-2}+\frac{D}{x+1}+\frac{E}{(x+2)^2}+\frac{F}{(x+1)^2}$$