Review

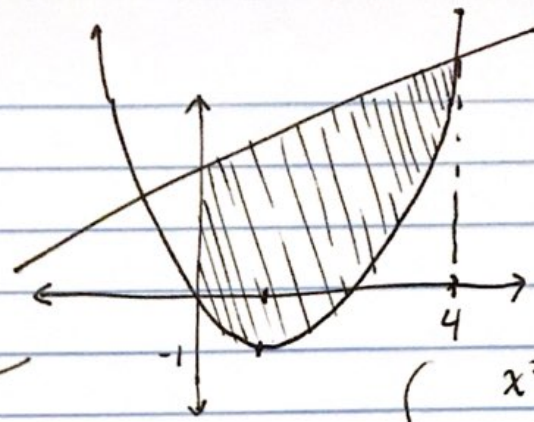
1) $y = x^2 - 2x$

$y' = 2x - 2 = 0$

$2x = 2$

$x = 1$

$\underbrace{\hspace{6em}}_{\text{sketch (optional)}}$




Find intersection
$$\left\{\begin{aligned} x^2 - 2x &= x + 4 \\ x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \\ x &= 4,\ -1 \end{aligned}\right.$$

$$A = \int_0^4 (x+4) - (x^2 - 2x)\, dx$$

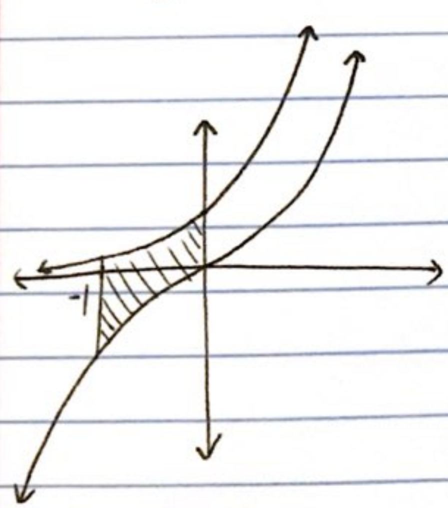
$$= \int_0^4 (x + 4 - x^2 + 2x)\, dx$$

$$= \int_0^4 (3x - x^2 + 4)\, dx$$

$$= \left[\frac{3}{2}x^2 - \frac{x^3}{3} + 4x\right]_0^4 = \left(24 - \frac{64}{3} + 16\right) - 0$$

$$= 40 - \frac{64}{3} = \boxed{\frac{56}{3}}$$

2)



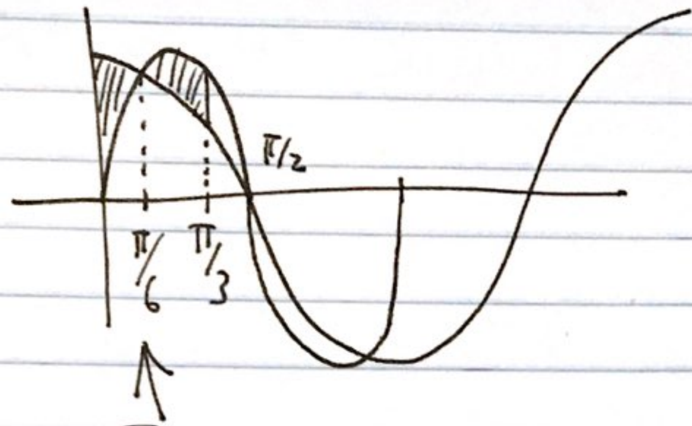

$$A = \int_{-1}^{0} (e^x - x^3)\, dx$$

$$= \left[e^x - \frac{x^4}{4}\right]_{-1}^{0}$$

$$= (1 - 0) - \left(e^{-1} - \frac{1}{4}\right)$$

$$= \boxed{\frac{5}{4} - \frac{1}{e}}$$

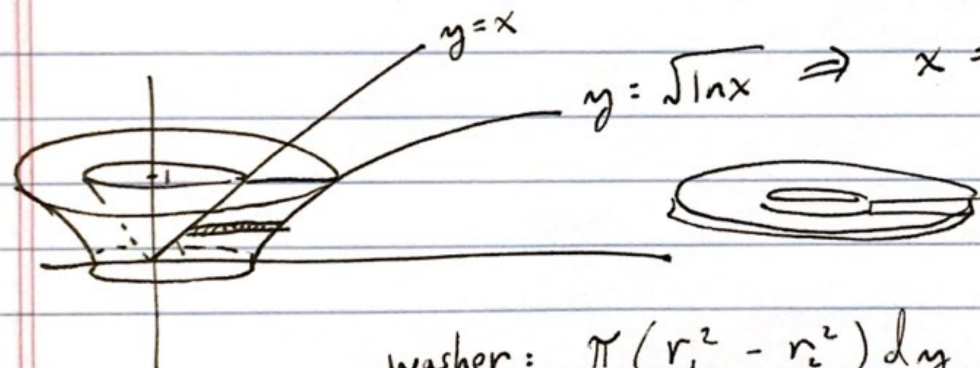
5)




$$\begin{aligned} \sin 2x &= \cos x \\ 2\sin x \cos x &= \cos x \\ 2\sin x &= 1 \\ \sin x &= \tfrac{1}{2} \\ x &= \pi/6 \end{aligned}$$

$$A = \int_0^{\pi/6} (\cos x - \sin 2x)\, dx + \int_{\pi/6}^{\pi/3} (\sin 2x - \cos x)\, dx$$

10)



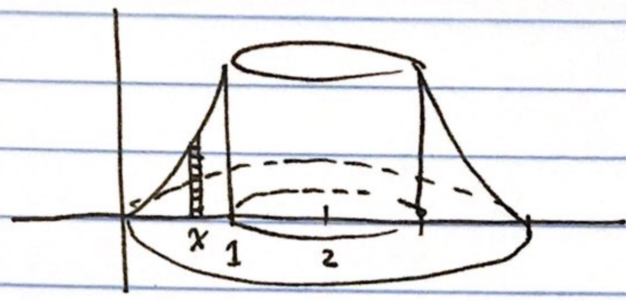

$y = \sqrt{\ln x} \Rightarrow x = e^{y^2}$

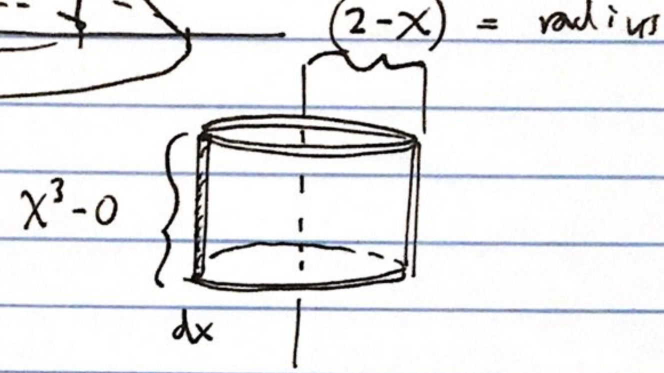
washer: $\pi(r_1^2 - r_2^2)\,dy$

$$V = \int_0^1 \pi\left(\left(e^{y^2}\right)^2 - y^2\right) dy$$

$$= \boxed{\int_0^1 \pi\left(e^{2y^2} - y^2\right) dy}$$

9)




$$dV = 2\pi(2-x)(x^3-0)\,dx$$

$$V = \int_0^1 2\pi\,(2-x)\,x^3\,dx \quad = \frac{3\pi}{5}$$

#14

$$\int \sin^7 x \cos^6 x \, dx$$

$$= \int (\sin^2 x)^3 \sin x \, \cos^6 x \, dx$$

$$= \int (1-\cos^2 x)^3 \sin x \, \cos^6 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= \int -(1-u^2)^3 u^6 \, du$$

$$= \int -(1 - 3u^2 + 3u^4 - u^6) u^6 \, du$$

$$= \int (-u^6 + 3u^8 - 3u^{10} + u^{12}) \, du$$

$$= \frac{-u^7}{7} + \frac{3u^9}{9} - \frac{3u^{11}}{11} + \frac{u^{13}}{13} + C$$

$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{3} - \frac{3\cos^{11} x}{11} + \frac{\cos^{13} x}{13} + C$$