Calc 2

Welcome to Day 1!

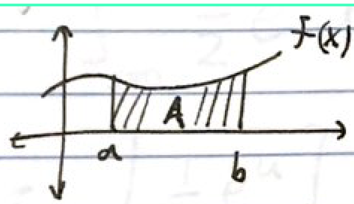
Course overview: 1) integration

→ new methods of integration

→ applications of integration

→ areas + volumes

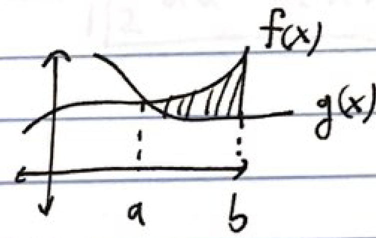
Recall the Fundamental Theorem of Calculus: the area under the curve increases at the rate equal to the curve height at x. So the derivative of the area function is the original function.



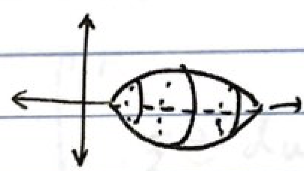
So, area = anti-derivative of f(x).

$$A = \int_a^b f(x)\,dx$$

A = F(b) -F(a), where F′=f

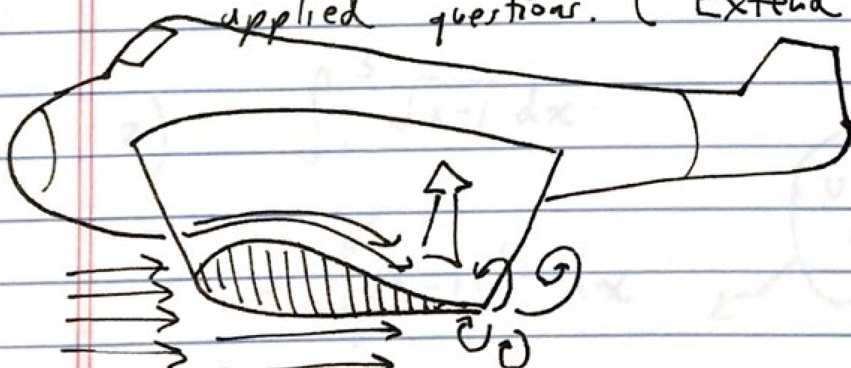


$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

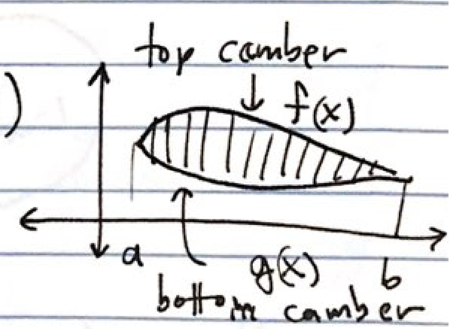


Volume

2) Sequences and series : used to get arbitrarily accurate answers to applied questions. (Extend linear approximation.)



1)



Lift = bottom pressure - top pressure + correction$_1$ - correction$_2$ + ... - ..

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx$$

6.1 Areas between curves.

Integral Review

1) $\int_0^2 x e^{x^2-3}\,dx$

$\boxed{u = x^2-3}$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\boxed{\frac{1}{2}du = x\,dx}$

$= \int_{x=0}^{x=2} \frac{1}{2} e^u\,du$

$= \left[\frac{1}{2}e^u\right]_{x=0}^{2}$

$= \left[\frac{1}{2}e^{x^2-3}\right]_0^2 = \frac{1}{2}e^{4-3} - \frac{1}{2}e^{0-3} = \frac{1}{2}\left(e - \frac{1}{e^3}\right)$

OR $= \int_{-3}^{1} \frac{1}{2}e^u\,du = \left[\frac{1}{2}e^u\right]_{-3}^{1} = \frac{1}{2}\left(e - \frac{1}{e^3}\right)$

x	u
0	-3
2	1

2) $\int_1^5 \sqrt{x-1}\,dx$

use $u = x-1$

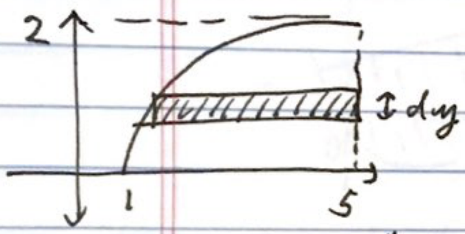
$u = x - 1$

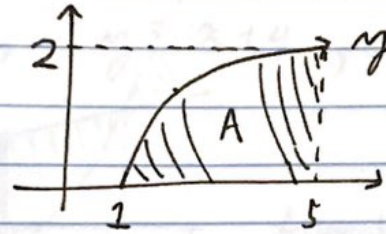
$du = dx$

$= \int_1^5 (x-1)^{1/2}\,dx$

$= \left[\frac{2}{3}(x-1)^{3/2}\right]_1^5 = \frac{2}{3}\left(4^{3/2} - 0\right) = \frac{16}{3}$

→ Alternate method:





$$y = \sqrt{x-1}$$
$$\Downarrow$$
$$y^2 = x-1$$
$$y^2 + 1 = x$$

Solve for x, and figure out the upper and lower y

$$A = \int_0^2 5\,dy - \int_0^2 (y^2+1)\,dy$$

$$= 10 - \left[\frac{y^3}{3} + y\right]_0^2 = \boxed{\frac{16}{3}}$$

OR $$\int_0^2 5 - (y^2+1)\,dy$$

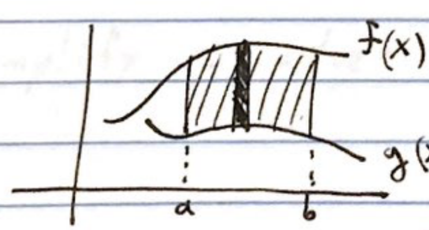
$$= \int_0^2 (4 - y^2)\,dy = \left[4y - \frac{y^3}{3}\right]_0^2$$

$$= 8 - \frac{8}{3} = \boxed{\frac{16}{3}}.$$

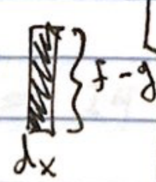
6.1 cont. Area between curves.

Two types of area:

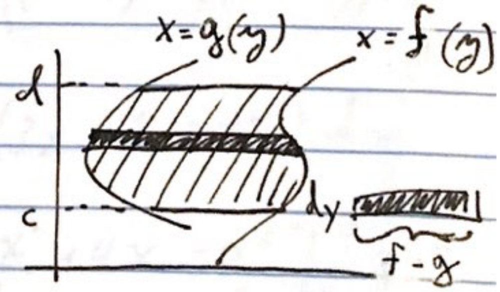
I.



$$A = \int_a^b \big(f(x) - g(x)\big)\,dx$$



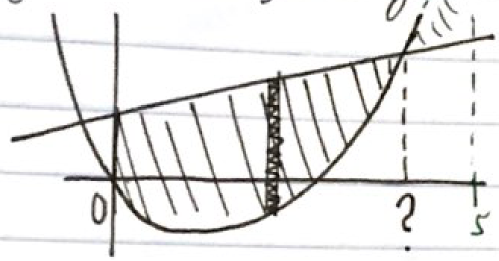
II.



$$A = \int_c^d \big(f(y) - g(y)\big)\,dy$$

Example Find the area enclosed between $y = x+4$ and $y = x^2-2x$, for $x > 0$.

$y = x^2 - 2x$, $y = x+4$, $a = 0$, $b = $ intersection



Steps 1) decide how to "slice" : type I

(since the top and bottom of the slice are the two functions for every slice.)

2) find missing b, the intersection:

Set equal: $x^2 - 2x = x + 4$

$\Rightarrow x^2 - 3x - 4 = 0$

$\Rightarrow (x-4)(x+1) = 0$

$\boxed{x = 4}$, $x = -1$

top bottom

3) Set up integral: $A = \int_0^4 \left((x+4) - (x^2 - 2x)\right) dx$

4) Simplify + solve: $A = \int_0^4 (3x + 4 - x^2)\, dx$

$= \left[\frac{3x^2}{2} + 4x - \frac{x^3}{3}\right]_0^4$

$= 24 + 16 - \frac{64}{3} = 40 - \frac{64}{3} = \boxed{\frac{56}{3}}$