Calculus II. Review for Test 3 with answers.

1. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

A: Limit $= 0 < 1$, so converges absolutely.

(b) $\sum_{n=1}^{\infty} \frac{2^{(n^2)}}{(2n)!}$

A: Limit $=\infty > 1$, so diverges.

2. For each series, what does the root test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\sum_{n=1}^{\infty} \left(\frac{-2n}{3n+1}\right)^n$

A: Limit $= 2/3 < 1$, so converges absolutely.

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}\right)^n$

A: Limit $= 0 < 1$, so converges absolutely.

3. For this power series, determine the radius of convergence.

(a) $\sum_{n=1}^{\infty} \frac{2(x-3)^n}{n5^n}$

A: $R = 5$

4. For this power series centered at $a = 2$, the radius of convergence is found to be $R = 3$. Determine the interval of convergence. (All you have to do is check the endpoints!)

(a) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^{(n+1)}}$

A: $[-1, 5)$

5. Find a power series for the following functions, by starting with a fact from the list of known Maclaurin series. Simplify just enough to combine the powers of x into a single expression.

(a) $f(x) = e^{-x}$

A: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

(b) $f(x) = x^2 \sin(2x)$

A: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{(2n+1)} x^{(2n+3)}}{(2n+1)!}$

(c) $f(x) = \dfrac{2x}{(1-x)^2}$

A: $\displaystyle\sum_{n=0}^{\infty} 2nx^n$

(d) $f(x) = \ln(1+2x)$

A: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{(n+1)} x^{(n+1)}}{n+1}$

(e) $f(x) = \dfrac{3}{2-x}$

A: $\displaystyle\sum_{n=0}^{\infty} \frac{3x^n}{2^{(n+1)}}$

6. Given $C = \begin{cases} x = t^2 + 1 \\ y = e^{(t+2)} - t \end{cases} \quad t \in [-3, 7]$.
Also given is that $y' = \frac{e^{(t+2)}-1}{2t}$.

(a) Find the (x, y) point(s) with horizontal tangent to the curve and use the second derivative to tell whether they are mins, maxes or inconclusive.

A: $(x, y) = (5, 3)$. At this point $y'' = \frac{1}{16} > 0$, so this point is a min.

(b) Set up the integral for the arc length of the curve.

$$A : \int_{-3}^{7} \sqrt{4t^2 + (e^{(t+2)} - 1)^2} dt$$

7. Given $C = \begin{cases} x = 1 - 2\cos t \\ y = -2 - \sin t \end{cases}$ for $t \in [\pi/2, 3\pi/2]$.

(a) Find the Cartesian xy-equation that the curve obeys: eliminate t.

$$A : \frac{(x-1)^2}{4} + (y+2)^2 = 1.$$

(b) Sketch C. Use arrows and label the points for the first and last t-values.

(answer shown in class.)

8. For the polar point $(r, \theta) = (-3, 7\pi/6)$, find the (x, y) coordinates. Graph the point.

A: $(x, y) = (\frac{3\sqrt{3}}{2}, \frac{3}{2})$ (Graph shown in class.)

9. For the point $(x, y) = (-5\sqrt{2}, 5\sqrt{2})$ find the polar coordinates with positive radius r.

A: $(r, \theta) = (10, \frac{3\pi}{4})$.

10. Graph the polar plot of $r = 1 - \sin\theta$.

A: (answer shown in class.)

11. Find the $n = 3$ term of the Taylor series for $f(x) = x^3 + e^{2x}$ centered at $a = 5$.

$$A : \frac{(6 + 8e^{10})(x-5)^3}{3!}$$

12. Also study the quizzes, and the homework questions. These are good test questions too!