Calculus II. Review for Test 3.

1. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{(n^2)}}{(2n)!}$

2. For each series, what does the root test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-2n}{3n+1}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}\right)^n$

3. For this power series, determine the radius of convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2(x-3)^n}{n5^n}$

4. For this power series centered at $a = 2$, the radius of convergence is found to be $R = 3$. Determine the interval of convergence. (All you have to do is check the endpoints!)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^{(n+1)}}$

5. Find a power series for the following functions, by starting with a fact from the list of known Maclaurin series. Simplify just enough to combine the powers of x into a single expression.

   (a) $f(x) = e^{-x}$

   (b) $f(x) = x^2 \sin(2x)$

   (c) $f(x) = \dfrac{2x}{(1-x)^2}$

   (d) $f(x) = \ln(1 + 2x)$

   (e) $f(x) = \dfrac{3}{2-x}$

6. Given $C = \begin{cases} x = t^2 + 1 \\ y = e^{(t+2)} - t \end{cases}$ $t \in [-3, 7]$.

   Also given is that $y' = \dfrac{e^{(t+2)}-1}{2t}$.

   (a) Find the (x, y) point(s) with horizontal tangent to the curve and use the second derivative to tell whether they are mins, maxes or inconclusive.

   (b) Set up the integral for the arc length of the curve.

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7. Given $C = \begin{cases} x = 1 - 2\cos t \\ y = -2 - \sin t \end{cases}$ for $t \in [\pi/2, 3\pi/2]$.

(a) Find the Cartesian xy-equation that the curve obeys: eliminate t.

(b) Sketch C. Use arrows and label the points for the first and last t-values.

8. For the polar point $(r, \theta) = (-3, 7\pi/6)$, find the (x, y) coordinates. Graph the point.

9. For the point $(x, y) = (-5\sqrt{2}, 5\sqrt{2})$ find the polar coordinates with positive radius r.

10. Graph the polar plot of $r = 1 - \sin\theta$.

11. Find the $n = 3$ term of the Taylor series for $f(x) = x^3 + e^{2x}$ centered at $a = 5$.

12. Also study the quizzes, and the homework questions. These are good test questions too!