Calculus II. Review for Test 2.

1. Find these definite integrals and classify as "divergent" or "convergent":

   a) $\frac{1}{2e^9}$ *(converges)*

   b) $-\infty$ *(diverges)* (Hint: it's $-\infty$ because you take $\lim_{t\to 0^-}$.)

   c) $\frac{32}{3}$ *(converges)*

2. For each of these sequences, find the limits, if they exist, and decide "diverges" or "converges."

   a) 0 *(converges)* (Hint: $(2/3)^n$ goes to zero since 2/3 is a fraction smaller than 1.)

   b) 0 *(converges)*

   c) 1/7 *(converges)*

   d) *DNE* *(diverges)* (Hint: $\cos(n\pi) = (-1)^n$ just from looking at the graph of $\cos x$.)

   e) $\pi/6$ *(converges)*

   f) 0 *(converges)*

   g) *DNE* *(diverges)*

3. For each series, what does the limit test for divergence tell us? [converge, diverge, or inconclusive] Show your work by performing the test.

   a) *diverges* (since limit $= 1/5$.)

   b) *inconclusive* (since limit $= 0$.)

   c) *inconclusive* (since limit $= 0$.)

4. For each series, what does the geometric series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work, and find the value if it converges.

a) $converges: \quad \dfrac{3/\pi}{1-3/\pi}$

b) $diverges: \quad r = \dfrac{5}{\sqrt{3}} > 1$

c) $diverges: \quad r = 2 > 1$

d) $converges: \quad -1/3$

e) $converges: \quad 3\dfrac{1/(e^2)}{1-(1/(e^2))}$

f) NA (Hint: does not apply since there is no way to write as r^n.)

5. For each series, what does the p-series test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

   a) NA

   b) $diverges \quad p = 0.5 \leq 1$)

   c) $diverges \quad p = 1 \leq 1$

   d) $converges \quad p = 3 > 1$

6. For each series, what does the integral test tell us? [not applicable, converge, diverge, or inconclusive] Show your work.

   a) $converges: \quad \displaystyle\int_{x=1}^{\infty} \frac{\sqrt{x}+4}{x^2}dx = 6$

   b) $diverges: \quad \displaystyle\int_{x=1}^{\infty} \frac{1}{\sqrt{x+1}}dx = \infty$

   c) $converges: \quad \displaystyle\int_{x=1}^{\infty} \left(\frac{1}{x}\right)^3 dx = 1/2$

$d)$ NA (since not positive)

Note $\sum_{n=1}^{\infty} \frac{1}{2^n}$ does converge by the integral test since the integral is $\int_1^{\infty} 1/(2^x)dx = \frac{1}{2\ln 2}$.

$e)$ NA (since $1/\cos^2 x$ is not decreasing)

7. For each series, what does the comparison test tell us? [not applicable, converge, or diverge] Show your work.

   $a)$ $converges: \ \frac{1}{n^3} \geq \frac{1}{2n^3+1}$ (Hint: for comparison test and limit comparison we will always be comparing to a p-series or geometric series!)

   $b)$ $converges: \ \frac{9^n}{10^n} \geq \frac{9^n}{3+10^n}$

   $c)$ $diverges: \ \frac{6^n}{5^n} \leq \frac{6^n}{-4+5^n}$

   $d)$ NA (since not positive)

8. For each series, what does the limit comparison test tell us? [not applicable, converge, or diverge] Show your work.

   $a)$ $diverges: \ \lim_{n\to\infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = 1/2$

   $b)$ $converges: \ \lim_{n\to\infty} \frac{\frac{n+2}{(n+1)^3}}{\frac{1}{n^2}} = 1$

   $c)$ $converges: \ \lim_{n\to\infty} \frac{\frac{2^n}{5^n-n}}{\frac{2^n}{5^n}} = 1$

9. For each series, use the alternating series test or the limit test for divergence to decide: [converge, or diverge]. Show your work.

   $a)$ $converges: \ \lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$

   $b)$ $converges: \ \lim_{n\to\infty} \frac{1}{4n+1} = 0$

   $c)$ $diverges: \ \lim \frac{(-1)^n}{[illegible]} = DNE$ since $\lim \frac{1}{[illegible]} = \infty$

10. Decide if the sums converge or diverge, explain why. If there is a formula for the sum, find the value.

$a)$ $converges:$ $\int_{x=1}^{\infty} x^2 e^{-x^3}\,dx = \frac{1}{3e}$ (by integral test)

$b)$ $diverges:$ $\lim_{n\to\infty} e^{2n} = \infty$ (by limit test for divergence)

$c)$ $converges:$ $\frac{2/e^3}{1-2/e^3}$ (by geometric series test)

11. For each series, what does the ratio test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

A: Limit $= 0 < 1$, so converges absolutely.

(b) $\sum_{n=1}^{\infty} \frac{2^{(n^2)}}{(2n)!}$

A: Limit $=\infty > 1$, so diverges.

12. For each series, what does the root test tell us? [converge absolutely, diverge, or inconclusive]. Show your work by finding the limit.

(a) $\sum_{n=1}^{\infty} \left(\frac{-2n}{3n+1}\right)^n$

A: Limit $= 2/3 < 1$, so converges absolutely.

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}\right)^n$

A: Limit $= 0 < 1$, so converges absolutely.

13. For this power series, determine the radius of convergence.

(a) $\sum_{n=1}^{\infty} \frac{2(x-3)^n}{n5^n}$

A: $R = 5$

14. For this power series , the radius of convergence is found to be $R = 3$. Determine the interval of convergence.

(a) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^{(n+1)}}$

A: $[-1, 5)$

15. For this power series, determine the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{5^n x^n}{1+2^n}$$

A: $(\frac{-2}{5}, \frac{2}{5})$

16. Also study the quizzes, and the homework questions. These are good test questions too!