Calculus II. Review 1, answers.

Note: these answers are only for checking your work. An answer on the test must show all the steps for full credit, similar to the way I work them on the board in class!

Also study quizzes, homework, and examples from notes!

For each integration problem, you must show the set-up and all the steps.

1. Find the area between the curves $y = x^2 - 2x, y = x + 4$, and $x = 0$ for $x > 0$.

$$56/3$$

2. Find the area between $y = x^3, \quad y = e^x, \quad x = -1, \quad x = 0$.

$$\frac{5}{4} - \frac{1}{e}$$

3. Find the area between $y = x - 1$ and $y^2 = 2x + 6$.

$$18$$

4. (skipped)

5. Just set up the integral for the area between $y = \cos x$ and $y = \sin 2x$ for $0 \leq x \leq \pi/3$.

$$\int_0^{\pi/6} (\cos x - \sin 2x)dx + \int_{\pi/6}^{\pi/3} (\sin 2x - \cos x)dx$$

6. Find the volume of the region inside $x = 0, \quad y = 3x + 1, \quad x = 2, \quad x = y^2$ rotated around the x-axis.

$$\int_0^2 \pi((3x+1)^2 - x)dx = 36\pi$$

7. Find the volume of the region inside $x = 0, \quad x = 1, \quad y = 2x, \quad y = e^{x^2}$ rotated around the y-axis.

$$\int_0^1 2\pi x(e^{x^2} - 2x)dx = \pi(e - 7/3)$$

8. Just set up the integral for the volume of the region inside $x = 0, \quad x = 1, \quad y = 2x, \quad y = e^{x^2}$ rotated around the x-axis.

$$\int_0^1 \pi(e^{2x^2} - 4x^2)dx$$

9. Find the volume of the region inside $y = x^3, \quad y = 0, \quad x = 1$ rotated around the line $x = 2$.

$$\int_0^1 2\pi(2 - x)(x^3)dx = \frac{3\pi}{5}$$

10. Just set up the integral for the volume of the region bounded by:
$y = 0,\ y = 1,\ y = x,\ y = \sqrt{\ln(x)}$; rotated around the $y-$axis.

$$\int_0^1 \pi(e^{2y^2} - y^2)dy$$

11. Find the average value of the function $f(x) = \dfrac{x+7}{\sqrt{x}}$ on the interval $[0, 3]$.

$$\frac{16\sqrt{3}}{3}$$

12. Evaluate the definite integral. $\displaystyle\int_1^2 x^3 \ln(x)dx$

$$\ln(16) - 15/16$$

13. Find the indefinite integral. $\displaystyle\int e^x \sin(2x)dx$

$$\frac{1}{5}e^x(\sin(2x) - 2\cos(2x)) + c$$

14. Find the indefinite integral. $\displaystyle\int \sin^7 x \cos^6 x dx$

$$\frac{-\cos^7(x)}{7} + \frac{3\cos^9(x)}{9} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^{13}(x)}{13} + c$$

15. Find the indefinite integral. $\displaystyle\int \sin^8 x \cos^5 x dx$

$$\frac{\sin^9(x)}{9} - \frac{2\sin^{11}(x)}{11} + \frac{\sin^{13}(x)}{13} + c$$

16. Find the indefinite integral. $\displaystyle\int x^2 e^x dx$

$$e^x(x^2 - 2x + 2) + c$$

17. Find the indefinite integral. $\displaystyle\int \sqrt{16 - x^2}dx$

$$\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}(x/4) + c$$

18. Find the indefinite integral. $\displaystyle\int \frac{1}{x^2\sqrt{x^2 - 16}}dx \ = \ \frac{\sqrt{x^2 - 16}}{16x} + c$

19. Show the correct form for a partial fraction decomposition of these functions. Don't actually solve for the variables.

a) $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{x+2}$

b) $\dfrac{A}{x-2} + \dfrac{B}{x+2}$

c) $\dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

20. Decompose the function into its partial fractions. (Actually solve for the variables.)

a) $\dfrac{7/4}{x-1} + \dfrac{(-7/4)x + 21/4}{x^2+3}$

b) $\dfrac{1/5}{x-2} + \dfrac{-1/5}{x+3}$

21. Find the indefinite integrals:

a) $x + 3\ln|x| - 2\ln|1+x| + c$ (Hint: first do long division since the degree of the numerator and denominator are both 2. This will turn the integral into: $\int (1 + \frac{x+3}{x(x+1)} dx$.)

b) $\dfrac{-4}{3(x+1)} + \dfrac{11}{9}\ln|x-2| - \dfrac{11}{9}\ln|x+1| + c$