

Calculus I. Fall '22 Test 2 Review (selected answers).

Make sure you also study all the quizzes, then notes and homework examples!

1. (previous link)
2. (previous link)
3. The refraction of a crystal as time increases is given by  $R = 23 - t^2 - 5 \cos(t - 4)$ . At  $t = 4$  is  $R$  increasing or decreasing, and is that change speeding up or slowing down?

$$R'(4) = -8 \text{ decreasing}$$

$$R''(4) = 3 \text{ slowing down}$$

4. Find the tangent line to the curve given by  $xy + 4y = 1 + 3e^x$  at  $(x, y) = (0, 1)$ .

$$y - 1 = \frac{1}{2}x$$

5. Find the linearization  $L(x)$  to  $f(x) = x^3 + 4x$  at  $x_1 = 1$ . Use it to approximate  $f(1.01)$ .

$$L(x) = 7(x - 1) + 5; L(1.01) = 5.07$$

6. A particle is moving along the curve given by  $xy + 1 = 2y^3e^{(x-1)}$ . At the point  $(1,1)$  the  $x$ -coordinate is increasing at the rate 5 m/s. Find the rate of change in the  $y$ -coordinate.

$$y' = -1$$

7. A rocket is descending straight down to rendezvous with a drone ship. The rocket velocity is -5 ft/s. The drone ship is heading west to reach the rendezvous point, decreasing its distance, at -10 ft/s. What is the rate of change in the straight line distance between them when the rocket is 40 ft above, and the drone is 30 ft east, of the rendezvous point?

$$d' = -10 \text{ ft/s}$$

8. Find critical numbers.

a)  $f(x) = x^{(4/5)}(x - 4)^2$

$$x = 0, 4, 8/7$$

b)  $f(x) = x^2e^{-3x}$

$$x = 0, 2/3$$

c)  $f(x) = x^{-2} \ln x$

$$x = \sqrt{e}$$

d)  $f(x) = \frac{1}{x} + 3x$

$$x = \sqrt{1/3}, -\sqrt{1/3}$$

e)  $f'(x) = 2 \cos(3x) + 1; 0 < x < \pi.$

$$x = 2\pi/9, 4\pi/9, 8\pi/9$$

9. Find local min and/or max. Use the first derivative test.

a)  $f(x) = x^4 e^{-x}$

local min at  $x = 0$  local max at  $x = 4$

b)  $f(x) = x^5 - x^4$

local max at  $x = 0$  local min at  $x = 4/5$

c)  $f(x) = x^4 - 4x^3$

local min at  $x = 3$  neither at  $x = 0$

10. Find local min and/or max (just  $x$ -values). Use the second derivative test.

a)  $f(x) = \frac{1}{3}x^3 - x$

local min at  $x = 1$  local max at  $x = -1$

11. Find absolute min and max on the given interval.

$$f(x) = x + \frac{1}{x}, \quad \left[\frac{1}{5}, 4\right]$$

abs. min  $y = 2$  at  $x = 1$ ; abs. max  $y = 26/5$  at  $x = 1/5$ .

12. Find all inflection points.

a)  $f(x) = x^4$ .

none

b)  $f(x) = x^5 - x^4$ .

only  $x = 3/5$

c)  $f(x) = x^4 - 4x^3$ .

both  $x = 0, 2$

13. Find limits.

a)  $\lim_{x \rightarrow 2} \frac{e^{(x^3)} - e^{(4x)}}{x - 2}$

$$8e^8$$

b)  $\lim_{x \rightarrow 2} \frac{3x - 12}{2x - 1}$

$$-2$$

c)  $\lim_{x \rightarrow 2} \frac{\sin x}{(x - 2)^4}$

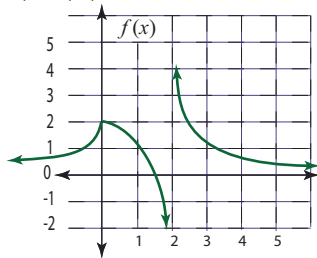
$$\infty$$

d)  $\lim_{x \rightarrow 0} \frac{e^x - e^{3x}}{1 - e^{2x}}$

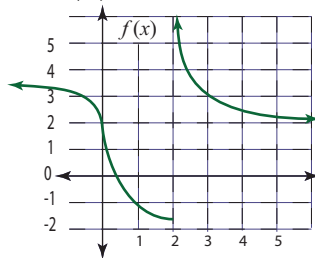
$$1$$

14. Choose the function graph that is possible given the (partial) information.

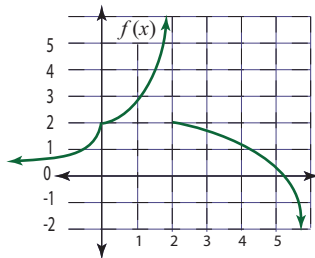
a)  $f'(x) > 0$  for  $0 < x < 2$ ;  $f''(x) > 0$  for  $0 < x < 2$



A.



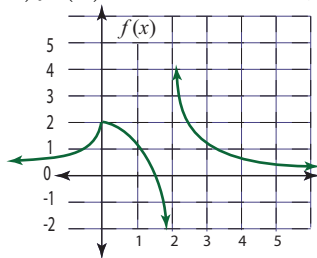
B.



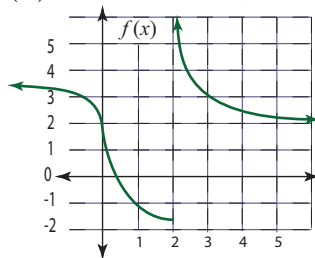
C.

Answer: C

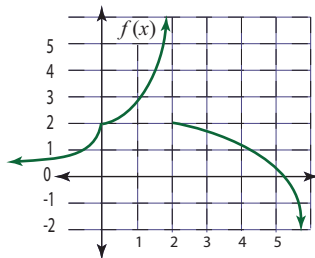
b)  $f'(x) < 0$  for  $x > 2$ ;  $f'(x) > 0$  for  $x < 0$ ; local max at  $x = 0$ .



A.



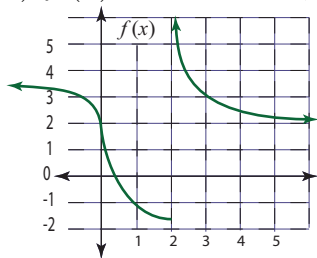
B.



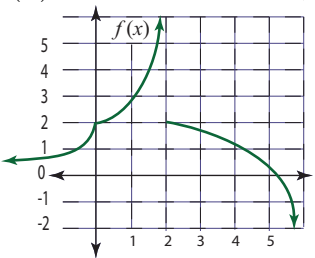
C.

Answer: A

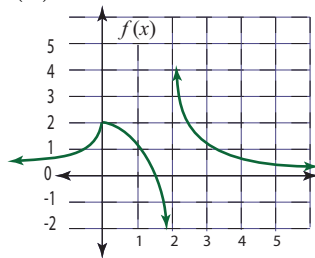
c)  $f'(x) < 0$  for  $x > 2$ ;  $f''(x) > 0$  for  $0 < x < 2$ ;  $f''(x) < 0$  for  $x < 0$ .



A.



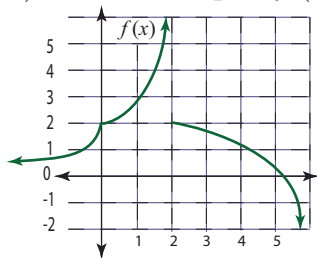
B.



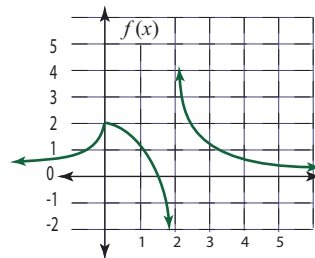
C.

Answer: A

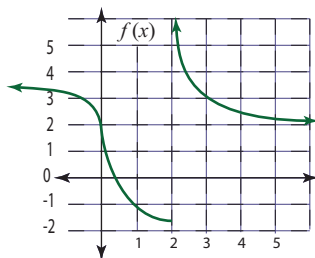
d)  $x = 0$  is an i.p.;  $f'(x) > 0$  for  $x < 0$ .



A.



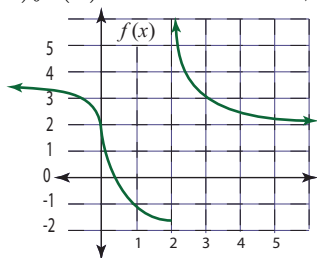
B.



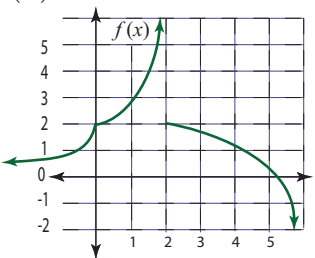
C.

Answer: B

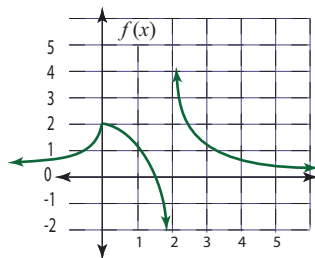
e)  $f'(x) < 0$  for  $x < 2$ ;  $f''(x) < 0$  for  $x < 0$ .



A.



B.



C.

Answer: A