Calculus I. Fall '22 Test 2 Review.
Make sure you also study all the quizzes, the derivative handout, then notes and homework examples!

1. Short derivatives. These are just for quick review; they may be seen as part of a test question.

   Power Rule:
   $y = x^2$

   $y = 7x^{-3}$

   $y = \sqrt[5]{x^7}$

   $y = x^{\sqrt{3}}$

   Exponential:
   $y = e^x$

   $y = 3^x$

   $y = (\ln 2)^x$

   Logs:
   $y = \ln x$

   $y = \log_5 x$

   $y = \log_{2\pi} x$

   Find $y' = \frac{dy}{dx}$ for these functions and relations involving: sums, products, quotients, compositions.

   You may need to use implicit differentiation and/or logarithmic differentiation.

2. Find y'. Don't simplify.

   a) $y = \dfrac{x^4 - \sqrt{x}}{\sin 3x}$

   b) $y = \dfrac{1}{\sqrt[7]{x^5}}$

   c) $y = e^x \cos^3(2^x)$

d) $y = \sec(\log_2(x))$

e) $y = \dfrac{\tan x}{e^x - \sqrt{x}}$

f) $x3^y = (x+1)y$

g) $xy = \csc y$

h) $y = x^{\left(\frac{5}{x}\right)}$

i) $y = \sin(x^{\left(\frac{5}{x}\right)})$

j) $y = \sin^{-1}(2^r)$

k) $y = \cos^{-1}(3^x \sin x)$

l) $y = x + 3^y$

m) $y^y = (x-y)^x$

n) $y = \dfrac{x+1}{1+x^2 e^x}$

o) $y = x^5 e^x 5^x$

p) $y = \sec(e^x 5^x) \tan x^2$

q) $y = \sec(5x+7) \tan^2 x$

r) $y = 2^{(\tan^{-1} 4x)}$

s) $y = \log_3 2x \log_7 5x$

t) $y = 7^{(\ln(2x+1))}$

u) $y = 7^x \ln(2x+1)$

v) $xy^2 = yx^3 + 1$

3. The refraction of a crystal as time increases is given by $R = 23 - t^2 - 5\cos(t-4)$. At $t = 4$ is R increasing or decreasing, and is that change speeding up or slowing down?

4. Find the tangent line to the curve given by $xy + 4y = 1 + 3e^x$ at $(x, y) = (0, 1)$.

5. Find the linearization $L(x)$ to $f(x) = x^3 + 4x$ at $x_1 = 1$. Use it to approximate $f(1.01)$.

6. A particle is moving along the curve given by $xy + 1 = 2y^3 e^{(x-1)}$. At the point $(1,1)$ the x-coordinate is increasing at the rate 5 m/s. Find the rate of change in the y-coordinate.

7. A rocket is descending straight down to rendezvous with a drone ship. The rocket velocity is -5 ft/s. The drone ship is heading west to reach the rendezvous point, decreasing its distance, at -10 ft/s. What is the rate of change in the straight line distance between them when the rocket is 40 ft above, and the drone is 30 ft east, of the rendezvous point?

8. Find all critical numbers (just the x-value).

a) $f(x) = x^{(4/5)}(x-4)^2$

b) $f(x) = x^2 e^{-3x}$

c) $f(x) = x^{-2} \ln x$

d) $f(x) = \dfrac{1}{x} + 3x$

e) $f(x) = x + \frac{2}{3}\sin(3x)$, and $f'(x) = 1 + 2\cos(3x)$; $x \in [0, \pi]$.

9. Find local min and/or max. Use the first derivative test.

a) $f(x) = x^4 e^{-x}$

b) $f(x) = x^5 - x^4$

c) $f(x) = x^4 - 4x^3$

10. Find local min and/or max. Use the second derivative test.

a) $f(x) = \dfrac{1}{3}x^3 - x$

11. Find absolute min and max on the given interval.

$$f(x) = x + \frac{1}{x}, \quad [\tfrac{1}{5}, 4]$$

12. Find all inflection points.

a) $f(x) = x^4$.

b) $f(x) = x^5 - x^4$.

c) $f(x) = x^4 - 4x^3$.

13. Find limits.

a) $\displaystyle \lim_{x \to 2} \frac{e^{(x^3)} - e^{(4x)}}{x - 2}$

b) $\displaystyle \lim_{x \to 2} \frac{3x - 12}{2x - 1}$

c) $\displaystyle \lim_{x \to 2} \frac{\sin x}{(x - 2)^4}$

d) $\displaystyle \lim_{x \to 0} \frac{e^x - e^{3x}}{1 - e^{2x}}$

14. Choose the function graph that is possible given the (partial) information.

a) $f'(x) > 0$ for $0 < x < 2$; $f''(x) > 0$ for $0 < x < 2$
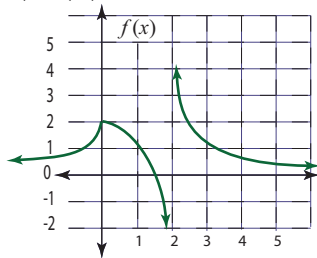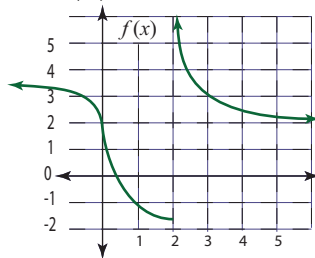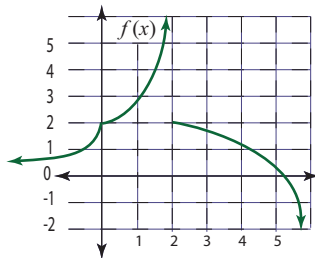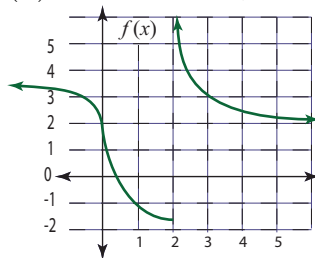

A.   B.   C.

Answer:_____

b) $f'(x) < 0$ for $x > 2$; $f'(x) > 0$ for $x < 0$; local max at $x = 0$.
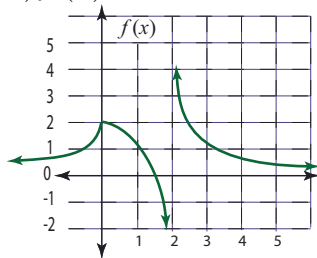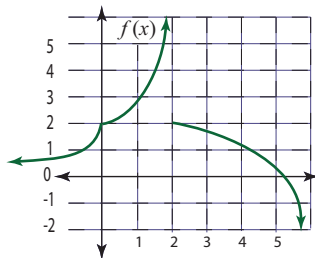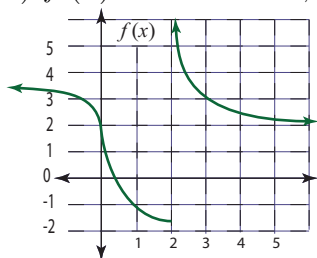

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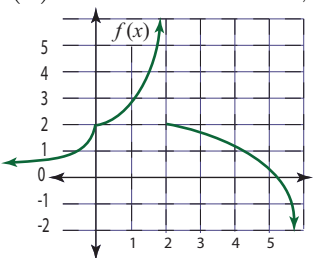
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c) $f'(x) < 0$ for $x > 2$; $f''(x) > 0$ for $0 < x < 2$; $f''(x) < 0$ for $x < 0$.
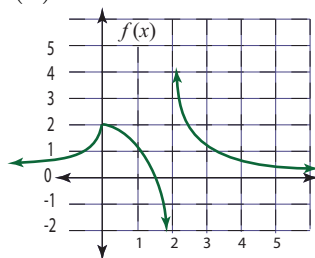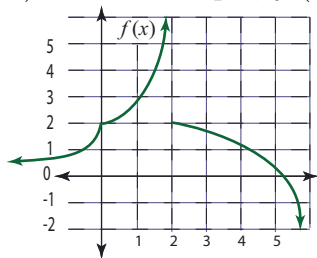

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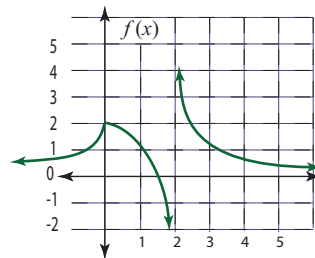
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d) $x = 0$ is an i.p. ; $f'(x) > 0$ for $x < 0$.
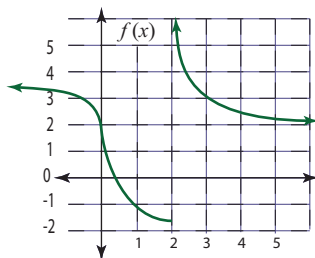
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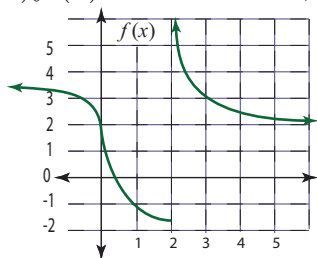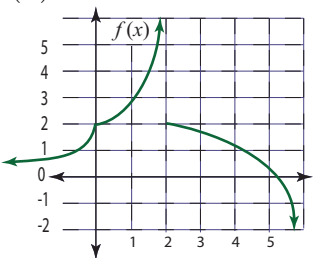
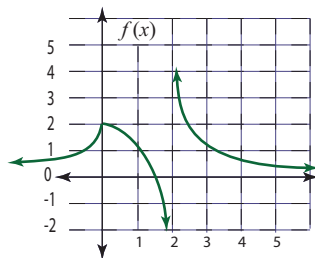Answer:_____

e) $f'(x) < 0$ for $x < 2$; $f''(x) < 0$ for $x < 0$.

A.   B.   C.

Answer:_____