Calculus I. Fall '19 Test 2 Review.
Make sure you also study all the quizzes, the derivative handout, then notes and homework examples!

1. Short derivatives. These are just for quick review; they may be seen as part of a test question.

Power Rule:

$y = x^2$ $\quad y' = \boxed{2x}$

$y = 7x^{-3}$ $\quad y' = \boxed{-21x^{-4}}$

$y = \sqrt[5]{x^7}$ $= x^{7/5}$ $\quad y' = \frac{7}{5}x^{2/5} = \boxed{\frac{7}{5}\sqrt[5]{x^2}}$

$y = x^{\sqrt{3}}$ $\quad y' = \boxed{\sqrt{3}\,x^{(\sqrt{3}-1)}}$

Exponential:

$y = e^x$ $\quad y' = \boxed{e^x}$

$y = 3^x$ $\quad y' = \boxed{3^x \ln 3}$

$y = (\ln 2)^x$ $\quad y' = \boxed{(\ln 2)^x \ln(\ln 2)} \approx (0.69)^x(-0.37)$

Logs:

$y = \ln x$ $\quad y' = \boxed{\frac{1}{x}}$

$y = \log_5 x$ $\quad y' = \boxed{\frac{1}{x \ln 5}}$

$y = \log_{2\pi} x$ $\quad y' = \boxed{\frac{1}{x \ln 2\pi}} \approx \frac{1}{x(1.84)}$

Find $y' = \frac{dy}{dx}$ for these functions and relations involving: sums, products, quotients, compositions.

You may need to use implicit differentiation and/or logarithmic differentiation.

2. Find y'. Don't simplify.

a) $y = \dfrac{x^4 - \sqrt{x}}{\sin 3x}$

$$y' = \boxed{\frac{(\sin 3x)(4x^3 - \frac{1}{2}x^{-1/2}) - (x^4 - x^{1/2})(\cos 3x)\cdot 3}{\sin^2 3x}}$$

b) $y = \dfrac{1}{\sqrt[7]{x^5}} = x^{-5/7}$

$$y' = \boxed{-\frac{5}{7}x^{-\frac{12}{7}}}$$

c) $y = e^x \cos^3(2^x)$

$$y' = \boxed{e^x(\cos^3(2^x)) + e^x(3\cos^2(2^x))(-\sin(2^x))2^x \ln 2}$$

parentheses!

d) $y = \sec(\log_2(x))$

$$y' = \boxed{\sec(\log_2 x)\tan(\log_2 x)\left(\frac{1}{x\ln 2}\right)}$$

e) $y = \dfrac{\tan x}{e^x - \sqrt{x}}$

$$y' = \boxed{\frac{(e^x - x^{1/2})\sec^2 x - \tan x\,(e^x - \frac{1}{2}x^{-1/2})}{(e^x - x^{1/2})^2}}$$

f) $x3^y = (x+1)y$

$$3^y + x3^y\ln 3\, y' = y + (x+1)y' \Rightarrow y' = \boxed{\frac{y - 3^y}{x3^y\ln 3 - (x+1)}}$$

g) $xy = \csc y$

$$y + xy' = (-\csc y\cot y)y' \Rightarrow y' = \boxed{\frac{-y}{x + \csc y\cot y}}$$

h) $y = x^{(\frac{5}{x})}$

$$\ln y = \frac{5}{x}\ln x \Rightarrow \frac{1}{y}y' = \frac{-5}{x^2}\ln x + \frac{5}{x}\left(\frac{1}{x}\right) \Rightarrow y' = \boxed{x^{\frac{5}{x}}\left(\frac{5}{x^2}\right)(1 - \ln x)}$$

i) $y = \sin(x^{(\frac{5}{x})})$

Use part(h)

$$y' = \boxed{\cos(x^{5/x})\, x^{5/x}\left(\frac{5}{x^2}\right)(1 - \ln x)}$$

j) $y = \sin^{-1}(2^r)$

$$y' = \boxed{\frac{1}{\sqrt{1 - (2^r)^2}}(2^r\ln 2)}$$

k) $y = \cos^{-1}(3^x \sin x)$

$$y' = \boxed{\frac{-1}{\sqrt{1 - (3^x\sin x)^2}}(3^x\ln 3\cdot\sin x + 3^x\cos x)}$$

l) $y = x + 3^y$

$$y' = 1 + 3^y\ln 3\, y' \Rightarrow \boxed{y' = \frac{1}{1 - 3^y\ln 3}}$$

m) $y^y = (x-y)^x$

$$y\ln y = x\ln(x-y) \Rightarrow y'\ln y + y\frac{1}{y}y' = \ln(x-y) + x\left(\frac{1}{x-y}\right)(1-y')$$

$$\Rightarrow \quad y' = \boxed{\dfrac{\ln(x-y) + \dfrac{x}{x-y}}{\ln y + 1 + \dfrac{x}{x-y}}}$$

n) $y = \dfrac{x+1}{1+x^2e^x}$

$$y' = \boxed{\dfrac{(1+x^2e^x) \quad -(x+1)(2xe^x + x^2e^x)}{(1+x^2e^x)^2}}$$

o) $y = x^5e^x5^x$

$$y' = \boxed{5x^4e^x5^x + x^5e^x5^x \quad + \quad x^5e^x5^x\ln 5}$$

p) $y = \sec(e^x5^x)\tan x^2$

$$y' = \boxed{\sec(e^x5^x)\tan(e^x5^x)(e^x5^x + e^x5^x\ln 5)\tan x^2 + \sec(e^x5^x)\sec^2(x^2)(2x)}$$

q) $y = \sec(5x+7)\tan^2 x$

$$y' = \boxed{\sec(5x+7)\tan(5x+7)(5)\tan^2 x + \sec(5x+7)2\tan x\sec^2 x}$$

r) $y = 2^{(\tan^{-1}4x)}$

$$y' = \boxed{2^{(\tan^{-1}4x)}\ln 2\left(\dfrac{1}{1+(4x)^2}\right)\cdot 4}$$

s) $y = \log_3 2x \log_7 5x$

$$y' = \boxed{\left(\dfrac{1}{2x\ln 3}\right)(2)\log_7 5x \quad + \quad \log_3 2x\left(\dfrac{1}{5x\ln 7}\right)(5)}$$

t) $y = 7^{(\ln(2x+1))}$

$$y' = \boxed{7^{(\ln(2x+1))}\ln 7\left(\dfrac{1}{2x+1}\right)\cdot 2}$$

u) $y = 7^x\ln(2x+1)$

$$y' = \boxed{7^x\ln 7\ln(2x+1) \quad + \quad 7^x\left(\dfrac{1}{2x+1}\right)\cdot 2}$$

v) $xy^2 = yx^3 + 1$

$$y^2 + x2yy' = y'x^3 + y3x^2 \quad \Rightarrow \quad y' = \boxed{\dfrac{3yx^2 - y^2}{2xy - x^3}}$$