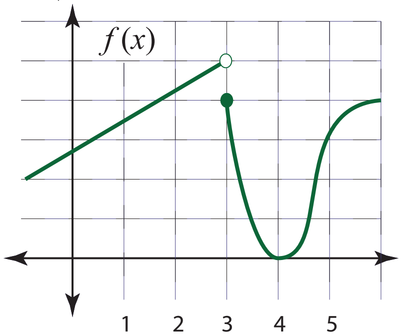
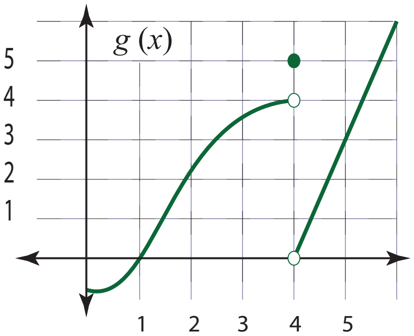
Calculus I. Fall Test 1 Review-Answers.
All trig and angles are in radians.
Make sure you also study all the quizzes, then notes and homework examples!

1. Use the graphs shown for f and g to evaluate each function value or limit (or answer DNE).





a) $f(3) = 4$

b) $g(4) = 5$

c) $\lim_{x \to 3^+} f(x) = 4$

d) $\lim_{x \to 3} f(x) = DNE$

e) $\lim_{x \to 4^-} [f(x) + g(x)] = 4$

f) $\lim_{x \to 3} \dfrac{f(x)}{g(x)} = DNE$

g) $\lim_{x \to 1} \dfrac{g(x)}{f(x)} = 0$

2.

$$\text{Given: } f(x) = \begin{cases} \frac{(7-x)}{3x^2-21x} & \text{for } x < 7 \\ 7x & \text{for } 7 \le x \end{cases}$$

a) $f(7) = 49$

b) $\lim_{x \to 7^+} f(x) = 49$

c) $\lim_{x \to 7^-} f(x) = \dfrac{-1}{21}$

d) $\lim_{x \to 7} f(x) = DNE$

e) Is $f(x)$ continuous at $x = 7$? If not, what kind of discontinuity is it ? No, it's a jump.

3. Find the following limits.

a) $\lim_{x\to 3} \frac{x^2+3x-1}{5-x} = \frac{17}{2}$

b) $\lim_{x\to 1} \frac{4x^2+3x-7}{2x-2} = \frac{11}{2}$

4. Find the following limits.

a) $\lim_{x\to\infty} \left(\frac{3x}{1-x} + e^{-\left(\frac{x^2+3x}{2x}\right)} \right) = -3$

b) $\lim_{x\to 0} \tan^{-1}\left(\frac{2x^3+4x}{10x^2+100x+57} \right) = 0$

c) $\lim_{x\to 4} \tan^{-1}\left(\frac{-1}{(x-4)^2} \right) = -\frac{\pi}{2}$

d) $\lim_{x\to\infty} \tan^{-1}\left(e^{\left(\frac{-1}{(x-4)^2}\right)} \right) = \frac{\pi}{4}$

5. If $f(x) = 5x + x^3$ then write the limit that will define $f'(x)$. (Just set it up, don't find the limit.)

$f'(x) = \lim_{h\to 0} \frac{5(x+h)+(x+h)^3-(5x+x^3)}{h}$

6. If $f(x) = 5 + x^{\sin(2x)}$ then write the limit that will define $f'(x)$. (Just set it up, don't find the limit.)

$f'(x) = \lim_{h\to 0} \frac{5+(x+h)^{(\sin(2(x+h)))}-(5+x^{\sin(2x)})}{h}$

7. $\lim_{h\to 0} \frac{(4(x+h)-3)-(4x-3)}{h} = 4.$

8. If $f'(5) = 7$ and $f(5) = 23$ then what is the equation of the tangent line to $f(x)$ at $x = 5$?

$\underline{y = 7x - 12}$

9. If $g(x) = \frac{x^3}{3} - x^2 + x$ and $g'(x) = x^2 - 2x + 1$, then find the equation of the tangent line to $g(x)$ at $x = -2$.

$\underline{y = 9x + \frac{28}{3}}$

10. Short derivatives. These are just for quick review; they may be seen as part of a test question. Find y' for each.

Power Rule:

$y = x^2$ $\qquad y' = 2x$

$y = 7x^{-3}$ $\qquad y' = -21x^{-4}$

$y = 2x + 1 - \dfrac{3}{x^2}$ $\qquad y' = 2 + 6x^{-3}$

$y = \sqrt[5]{x^7}$ $\qquad y' = \dfrac{7}{5}x^{2/5} = \dfrac{7\sqrt[5]{x^2}}{5}$

$y = x^{\sqrt{3}}$ $\qquad y' = \sqrt{3}x^{(\sqrt{3}-1)}$

Exponential:

$y = e^x$ $\qquad y' = e^x$

$y = 3^x$ $\qquad y' = 3^x \ln 3$

$y = (\ln 2)^x$ $\qquad y' = (\ln 2)^x \ln(\ln 2)$

Trig:

$y = \sin x$ $\qquad y' = \cos x$

$y = \cos x$ $\qquad y' = -\sin x$

$y = \tan x$ $\qquad y' = \sec^2 x$

$y = \cot x$ $\qquad y' = -\csc^2 x$

$y = \sec x$ $\qquad y' = \sec x \tan x$

$y = \csc x$ $\qquad y' = -\csc x \cot x$

11. Find y'. Don't simplify.

a) $y = \dfrac{x^4 - \sqrt{x}}{\sin x}$

$$y' = \frac{(\sin x)(4x^3 - \frac{1}{2}x^{-1/2}) - (x^4 - \sqrt{x})(\cos x)}{\sin^2 x}$$

b) $y = \dfrac{1}{\sqrt[7]{x^5}} = x^{-5/7}$ $\qquad y' = \dfrac{-5}{7}x^{-12/7}$

c) $y = x^e e^x$ $\qquad y' = ex^{(e-1)}e^x + x^e e^x$

d) $y = 3^x \sin x$ $\qquad y' = 3^x \ln 3 \sin x + 3^x \cos x$

e) $y = 7x^2 e^x \csc x$ $\qquad y' = 14xe^x \csc x + 7x^2(e^x \csc x - e^x \csc x \cot x)$

f) $y = 2^x \tan x$ $\qquad y' = 2^x \ln 2 \tan x + 2^x \sec^2 x$

g) $\dfrac{x+1}{1-\sin x}$ $\qquad y' = \dfrac{(1-\sin x) - (x+1)(-\cos x)}{(1-\sin x)^2}$

h) $\dfrac{x+2^x}{1-x^3e^x}$ $\qquad y' = \dfrac{(1-x^3e^x)(1+2^x \ln 2) - (x+2^x)(-3x^2e^x - x^3e^x)}{(1-x^3e^x)^2}$

i) $y = 7x \cot x$ $\qquad y' = 7 \cot x + 7x(-\csc^2 x)$

j) $y = \dfrac{\sec x}{x-1}$ $\qquad y' = \dfrac{(x-1)\sec x \tan x - \sec x}{(x-1)^2}$