week 2) ↓ Limit Rules & Methods

0)	When $f(x)$ is continuous, $\lim_{x \to x_1} f(x) = f(x_1)$	$\lim_{x \to 7} 4 = 4$
1)	When $f(x_1)$ gives $\frac{0}{0}$, try to factor and cancel the common factor before taking limit.	
2, 3, 4)	Combinations with $+, \times, \div$ can be done a piece at a time. (no division by 0)	
→ Piecewise $\left\{ \begin{array}{l} \cdot \\ \cdot \\ \cdot \end{array} \right.$	Make sure to use the correct formulas for $x \to x_1^+$, $x \to x_1^-$.	Match for total limit.
→ v.a.)	When $f(x_1)$ gives $\frac{\text{non zero}}{0}$, test x_1^- and x_1^+ by factoring (if possible) and compare sides of chart. If match, then $\boxed{\infty}$ or $\boxed{-\infty}$, no match means total limit $= \boxed{\text{DNE}}$	

week 3) ↓

→ v.a.) $b > 1$: $\lim_{x \to 0^+} \log_b x = -\infty$

h.a.) $f(x) = \frac{\text{root}}{\text{root}}$ → divide top and bottom by power of x

→ h.a.)

- For $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$,

degrees same: $\lim_{x \to \pm\infty} f(x) =$ ratio of high power coeffs.	deg. on top larger: $\lim_{x \to \pm\infty} f(x) = \pm\infty$ (chart!)
	degree on top smaller: $\lim_{x \to \pm\infty} f(x) = \boxed{0}$

→ h.a.)

$\lim_{x \to \infty} e^x = e^{\infty} = \boxed{\infty}$	$\lim_{x \to -\infty} e^x = e^{-\infty} = \boxed{0}$
$\lim_{x \to \infty} 2^x = 2^{\infty} = \boxed{\infty}$	$\lim_{x \to -\infty} 5^x = 5^{-\infty} = \boxed{0}$
$\lim_{x \to \infty} \tan^{-1} x = \tan^{-1} \infty = \boxed{\frac{\pi}{2}}$	$\lim_{x \to -\infty} \tan^{-1} x = \tan^{-1}(-\infty) = \boxed{\frac{-\pi}{2}}$

→ composition

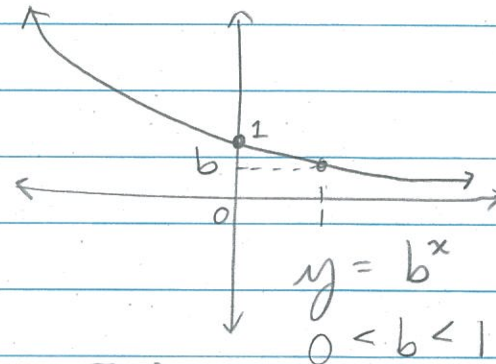
Ex: $\lim_{x \to \infty} \tan^{-1}\left(e^{(x^2/(1+x))}\right) = \tan^{-1}(e^{\infty}) = \tan^{-1} \infty = \boxed{\frac{\pi}{2}}$

Continuous functions include:

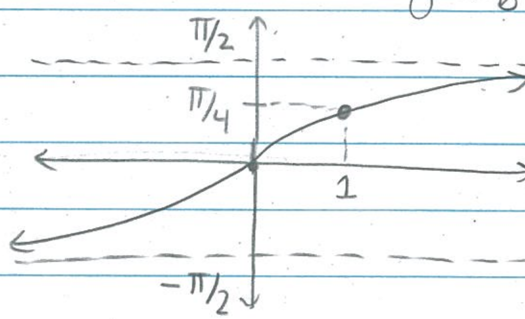
- $y = 3x + 7$; all lines
- $y = x^4 + 2x^3 - \frac{1}{2}x^2 + 5$; all polynomials
- $y = 7$; constants

$$\boxed{\lim_{x \to 10} 7 = 7}$$

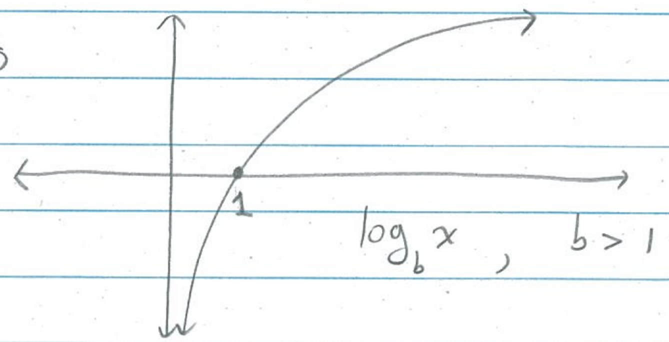
- $y = e^x$
- $y = 5^x$
- $y = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$



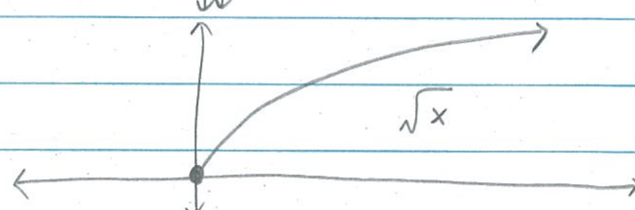
- $y = \sin x$
- $y = \cos x$
- $y = \tan^{-1} x = \arctan x$



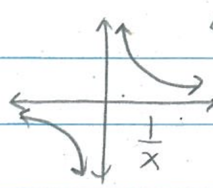
- $y = \ln x$, $x > 0$



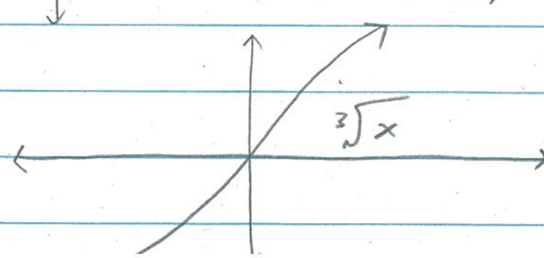
- $y = \sqrt{x}$, $x > 0$



- $y = \frac{1}{x}$, $x \neq 0$



- $y = \sqrt[3]{x} = x^{1/3}$



Rules for combining functions:

2) $$\lim_{x \to x_1} \left(f(x) + g(x) \right) = \left(\lim_{x \to x_1} f(x) \right) + \left(\lim_{x \to x_1} g(x) \right)$$

3) $$\lim_{x \to x_1} f(x)\, g(x) = \left(\lim_{x \to x_1} f(x) \right) \left(\lim_{x \to x_1} g(x) \right)$$

specifically

$$\lim_{x \to x_1} c f(x) = c \lim_{x \to x_1} f(x)$$

for c any real constant

4) $$\lim_{x \to x_1} \frac{f(x)}{g(x)}$$

$$= \frac{\lim_{x \to x_1} f(x)}{\lim_{x \to x_1} g(x)}$$

when $\lim_{x \to x_1} g(x) \neq 0$

Ex: Find limit

$$\lim_{x \to 0} \frac{x^{10}(x-3)e^{x}}{4x^{10}} \qquad \left(= \frac{0}{0}\right)$$

(cancel)

$$= \lim_{x \to 0} \frac{(x-3)e^{x}}{4}$$

$$= \frac{(0-3)e^{0}}{4} = \boxed{\frac{-3}{4}}$$

Ex: Find limit

$$\lim_{x \to 0} \frac{5(x-1)}{2(x-1)^{3}} \qquad \left(= \frac{0}{0}\right)$$

(cancel)

$$= \lim_{x \to 0} \frac{5}{2(x-1)^{2}} \qquad \left(= \frac{5}{0}\right)$$

$\frac{\text{nonzero}}{0} \Rightarrow$ v.a., so limit is ∞, $-\infty$, or DNE

test near $x_1 = 1$

	1^-	1	1^+
x	(0.9)		(1.1)
$f(x) = \frac{5}{2(x-1)^2}$	$\frac{+}{+}$		$\frac{+}{+}$
	$= +$		$= +$

matching positives, so answer $= \boxed{\infty}$