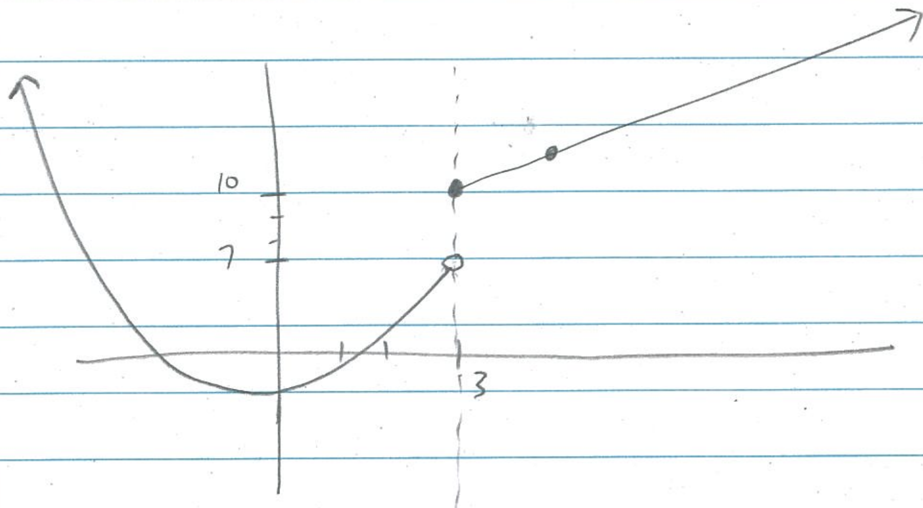
$$f(x) = \begin{cases} x+7, & x \geq 3 \\ x^2 - 2, & x < 3 \end{cases}$$



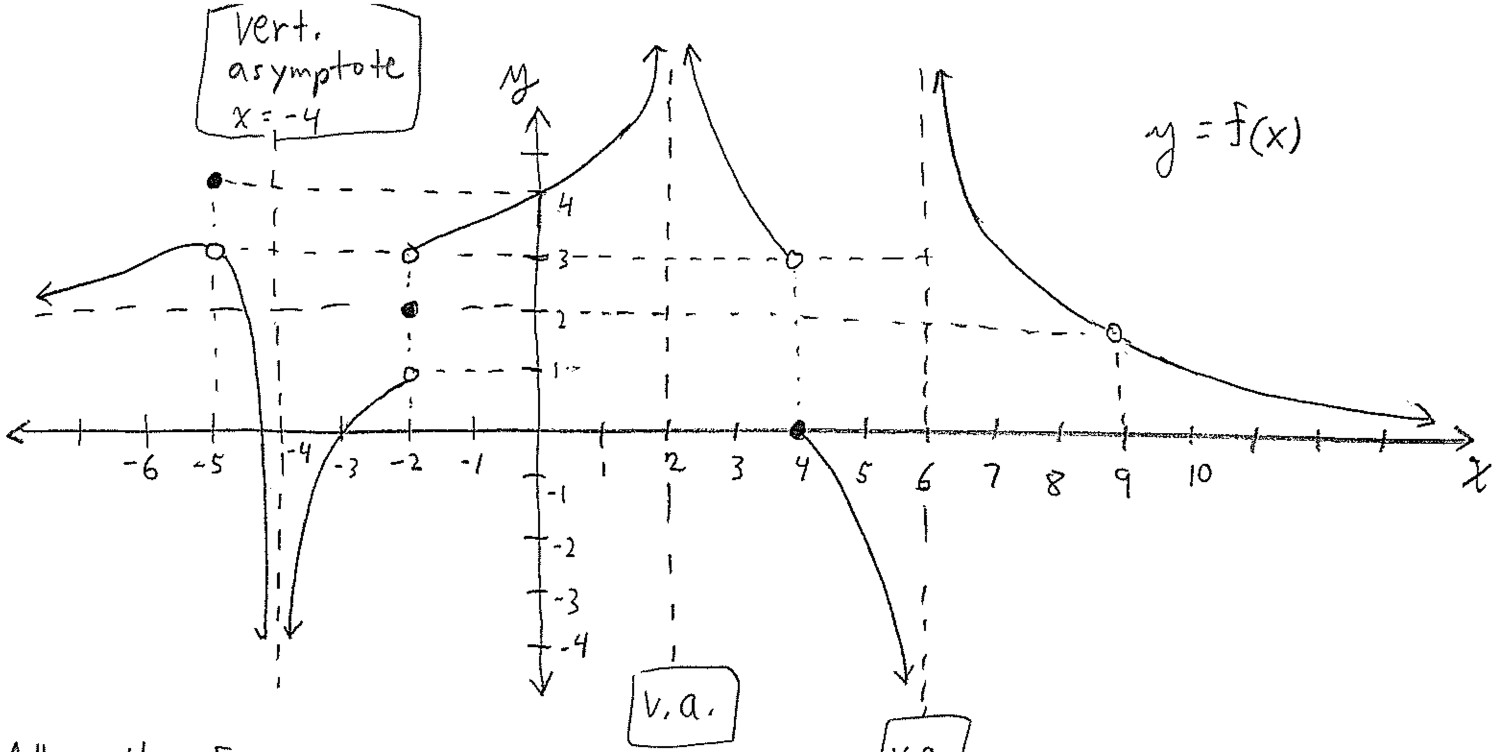
$f(3) = 10$
$f(4) = 11$
$f(1) = -1$

$$\lim_{x \to 3^-} f(x) = 7$$

$$\lim_{x \to 3^+} f(x) = 10$$

$$\lim_{x \to 3} f(x) = \text{DNE}$$

$$\left.\begin{array}{l} \displaystyle\lim_{x \to 4^-} f(x) = 11 \\ \displaystyle\lim_{x \to 4^+} f(x) = 11 \end{array}\right\} \quad \lim_{x \to 4} f(x) = \lim_{x \to 4} (x+7) = 4+7 = 11$$



All the facts:

$f(-5) =$ ____

$\lim_{x \to -5^-} f(x) =$ ____

$\lim_{x \to -5^+} f(x) =$ ____

$\lim_{x \to -5} f(x) =$ ____

$f(4) =$ ____

$\lim_{x \to 4^-} f(x) =$ ____

$\lim_{x \to 4^+} f(x) =$ ____

$\lim_{x \to 4} f(x) =$ ____

$f(-4) =$ ____

$\lim_{x \to -4^-} f(x) =$ ____

$\lim_{x \to -4^+} f(x) =$ ____

$\lim_{x \to -4} f(x) =$ ____

$f(-4) =$ ____

$\lim_{x \to -4^-} f(x) =$ ____

$\lim_{x \to -4^+} f(x) =$ ____

$\lim_{x \to -4} f(x) =$ ____

$f(9) =$ ____

$\lim_{x \to 9^-} f(x) =$ ____

$\lim_{x \to 9^+} f(x) =$ ____

$\lim_{x \to 9} f(x) =$ ____

$f(2) =$ ____

$\lim_{x \to 2^-} f(x) =$ ____

$\lim_{x \to 2^+} f(x) =$ ____

$\lim_{x \to 2} f(x) =$ ____

$f(-2) =$ ____

$\lim_{x \to -2^-} f(x) =$ ____

$\lim_{x \to -2^+} f(x) =$ ____

$\lim_{x \to -2} f(x) =$ ____

$f(0) =$ ____

$\lim_{x \to 0^-} f(x) =$ ____

$\lim_{x \to 0^+} f(x) =$ ____

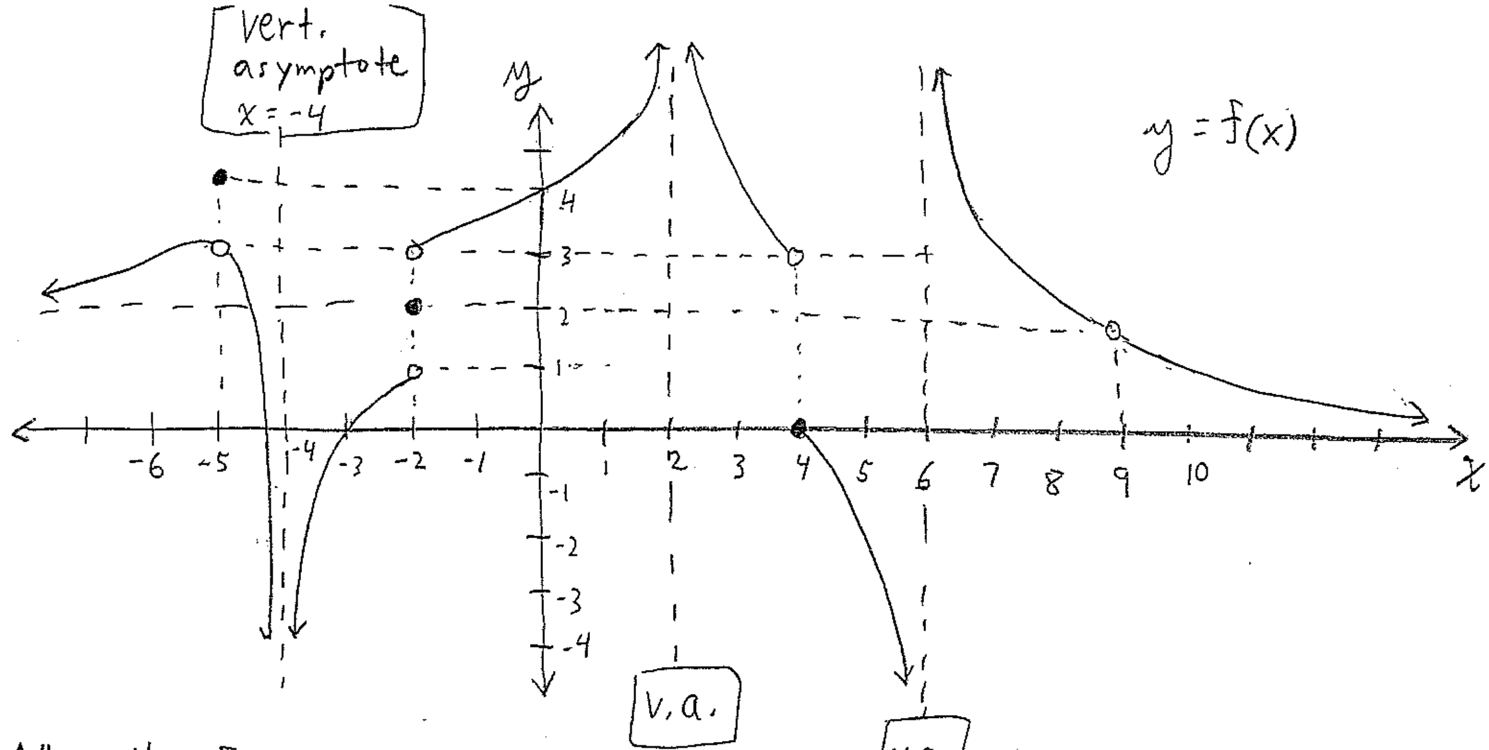
$\lim_{x \to 0} f(x) =$ ____

$f(6) =$ ____

$\lim_{x \to 6^-} f(x) =$ ____

$\lim_{x \to 6^+} f(x) =$ ____

$\lim_{x \to 6} f(x) =$ ____



All the facts:

$f(-5) = \underline{4}$

$\lim_{x \to -5^-} f(x) = \underline{3}$

$\lim_{x \to -5^+} f(x) = \underline{3}$

$\lim_{x \to -5} f(x) = \underline{3}$

$f(-4) = \underline{\text{DNE}}$

$\lim_{x \to -4^-} f(x) = \underline{-\infty}$

$\lim_{x \to -4^+} f(x) = \underline{-\infty}$

$\lim_{x \to -4} f(x) = \underline{-\infty}$

$f(-2) = \underline{2}$

$\lim_{x \to -2^-} f(x) = \underline{1}$

$\lim_{x \to -2^+} f(x) = \underline{3}$

$\lim_{x \to -2} f(x) = \underline{\text{DNE}}$

$f(4) = \underline{0}$

$\lim_{x \to 4^-} f(x) = \underline{3}$

$\lim_{x \to 4^+} f(x) = \underline{0}$

$\lim_{x \to 4} f(x) = \underline{\text{DNE}}$

$f(9) = \underline{\text{DNE}}$

$\lim_{x \to 9^-} f(x) = \underline{2}$

$\lim_{x \to 9^+} f(x) = \underline{2}$

$\lim_{x \to 9} f(x) = \underline{2}$

$f(0) = \underline{4}$

$\lim_{x \to 0^-} f(x) = \underline{4}$

$\lim_{x \to 0^+} f(x) = \underline{4}$

$\lim_{x \to 0} f(x) = \underline{4}$

$f(-4) = \underline{\text{DNE}}$

$\lim_{x \to -4^-} f(x) = \underline{-\infty}$

$\lim_{x \to -4^+} f(x) = \underline{-\infty}$

$\lim_{x \to -4} f(x) = \underline{-\infty}$

$f(2) = \underline{\text{DNE}}$

$\lim_{x \to 2^-} f(x) = \underline{\infty}$

$\lim_{x \to 2^+} f(x) = \underline{\infty}$

$\lim_{x \to 2} f(x) = \underline{\infty}$

$f(6) = \underline{\text{DNE}}$

$\lim_{x \to 6^-} f(x) = \underline{-\infty}$

$\lim_{x \to 6^+} f(x) = \underline{\infty}$

$\lim_{x \to 6} f(x) = \underline{\text{DNE}}$

Find
$$\lim_{x \to 9} \frac{3(x-9)}{x^2-81}$$

① Try plugging in $x = 9$

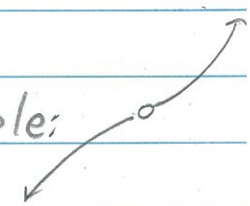
$$f(9) = \frac{3(9-9)}{81-81} = \frac{0}{0} = \text{DNE} \quad \left(\begin{matrix} \neq 1 \\ \neq 0 \end{matrix}\right)$$

$\Rightarrow$ discontinuous, [which occurs whenever you get: $\frac{\text{anything}}{0}$, $\sqrt{\text{negative}}$

or $\sqrt[4]{\text{negative}}$, or $\log_b 0$ or $\log_b(\text{negative})$.]

even root

[$\frac{0}{0}$ means it could either be a hole:



... or a vertical asymptote.]

② Try to cancel algebraically: since $x \to 9$ means $x \neq 9$, just getting close to 9.

$$= \lim_{x \to 9} \frac{3(x-9)}{(x-9)(x+9)}$$

$$= \lim_{x \to 9} \frac{3}{x+9}$$

Now do step ① again, and this time its continuous!

$$= \frac{3}{9+9} = \frac{3}{18} = \frac{1}{6}.$$ (This is a hole.)