Definitions for exponents

1) $x^3 = x \cdot x \cdot x \quad ; \quad x^1 = x$

2) $x^{3/8} = \sqrt[8]{x^3} \quad ; \quad x^{1/2} = \sqrt[2]{x} = \sqrt{x}$

3) $x^{-5} = \dfrac{1}{x^5} \quad ; \quad x^{-1/2} = \dfrac{1}{x^{1/2}} = \dfrac{1}{\sqrt{x}}$

for $x \neq 0$

4) $x^0 = 1 \quad ; \quad 1^x = 1$

for $x \neq 0$

Rules for exponents

1) $x^3 x^4 = x^7$

$x^{2/3} x^{-1} = x^{-1/3}$

5) $\left(\dfrac{x}{y}\right)^3 = \dfrac{x^3}{y^3}$

2) $(x^3)^4 = x^{12}$

$\left(x^{1/2}\right)^{-3/4} = x^{-3/8}$

3) $(xy)^3 = x^3 y^3$

4) $\dfrac{x^7}{x^3} = x^7 x^{-3} = x^{7-3} = x^4$

Ex: $4^{3/2} = \sqrt[2]{4^3} = \left(\sqrt[2]{4}\right)^3 = 2^3 = 8$

Defs for logs

1) $\log_3 x$ means "what number goes on base, 3, as exponent, to get x as a result?"

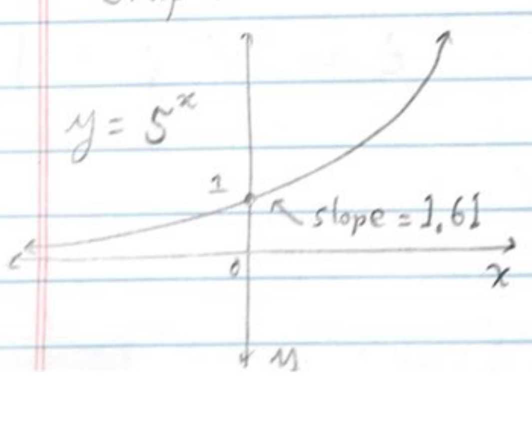
- $\log_3 3 = \boxed{1}$ since $3^1 = 3$.
- $\log_3 81 = \boxed{4}$ since $3^4 = 81$.
- $\log_9 3 = \boxed{\frac{1}{2}}$ since $9^{1/2} = 3$
- $\log_5\left(\frac{1}{25}\right) = \boxed{-2}$ since $5^{-2} = \frac{1}{25}$
- $\log_7 1 = \boxed{0}$ since $7^0 = 1$

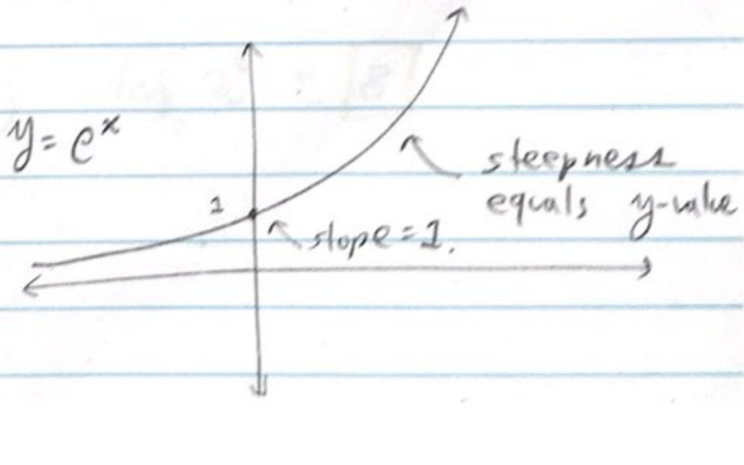
2) $\log_e x = \ln x$ "what number goes on $e = 2.718\ldots$ to get x?"

- $\ln e = \boxed{1}$
- $\ln 1 = \boxed{0}$
- $\ln e^{27} = \boxed{27}$

What is special about $e = 2.718\ldots$?

Graphs:





Rules for logs

1) $\log_x a + \log_x b = \log_x ab$

- $\log_3 5 + \log_3 2 = \boxed{\log_3 10}$

2) $\log_x a - \log_x b = \log_x \left(\frac{a}{b}\right)$

3) $\log_x a^b = b \log_x a$

- $\log_2 3^2 = 2 \log 3$
- $(\log_2 8)^2 = (\log_2 8)(\log_2 8) = 3 \log_2 8 = 3 \cdot 3 = \boxed{9}$
- $\log_2 8^2 = 2(\log_2 8) = 2 \cdot 3 = \boxed{6}$

4) $b^{\log_b x} = x$; $\log_b b^x = x$

- $e^{\ln 2} = 2$; $\ln e^7 = \boxed{7}$
- $7^{\log_7 5} = 5$; $\log_2 2^8 = \boxed{8}$

Examples: Simplify to have no logs.

1) Find $\log_7\left(\frac{1}{49}\right) - \log_3 3^{1/2}$

$= \log_7 1 - \log_7 49 - \frac{1}{2}$

$= 0 - 2 - \frac{1}{2}$

$= \boxed{-\frac{5}{2}}$

2) Solve for x:

$\log_2 3^x + 5x = 3 + x$

$\Rightarrow \quad x \log_2 3 + 4x = 3$

$\Rightarrow \quad x(\log_2 3 + 4) = 3$

$\Rightarrow \quad \boxed{x = \frac{3}{\log_2 3 + 4}}$

3) Simplify to have only one log.

$\ln 2x + 3\ln x + \log_2 2^x - \log_3 1$

$= \ln 2x + \ln x^3 + x - 0$

$= \boxed{\ln 2x^4 + x}$

Ex 4) Given $y = f(x) = \ln e^{x+2} - x^2 + \log_{\frac{1}{2}}(x+7)$

Find $f(-3)$

$$= \ln e^{-3+2} - (-3)^2 + \log_{\frac{1}{2}}(-3+7)$$

$$= \ln e^{-1} - 9 + \log_{\frac{1}{2}} 4$$

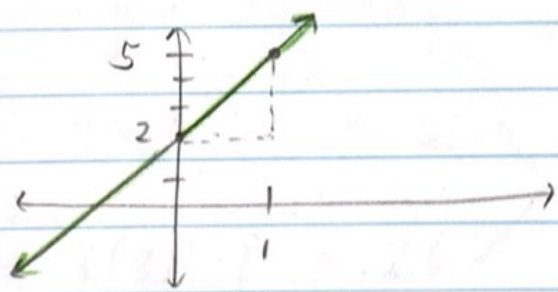
$$= -1 - 9 + -2$$

since $\left(\frac{1}{2}\right)^{-2} = \frac{1}{2^{-2}} = \frac{1}{1/4} = 4$

$$= \boxed{-12}$$

2.1 Secant lines

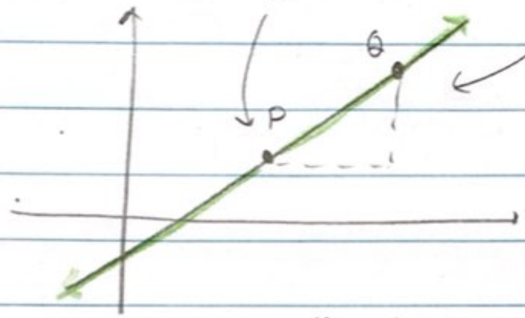
Recall $y = f(x) = 3x + 2$



$\boxed{y = mx + b}$, where $m = \frac{\text{rise}}{\text{run}}$

b = y-intercept

OR for two points on line
(x_1, y_1) and (x_2, y_2)



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

formula

$$\boxed{y - y_1 = m(x - x_1)}$$

(or use (x_2, y_2))

→ The secant line through $x_1 = 1$ and $x_2 = 3$ (points P and Q) of the function $y = 3x^2 - 1$.

$P: \quad y_1 = 3(1)^2 - 1 = 2$

$Q: \quad y_2 = 3(3)^2 - 1 = 26$

Slope of the secant line

$$m = \frac{26-2}{3-1} = \frac{24}{2} = \boxed{12}$$

⇒ secant line $\quad y - 2 = 12(x - 1)$

$\Rightarrow \quad y - 2 = 12x - 12$

$\Rightarrow \quad \boxed{y = 12x - 10}$