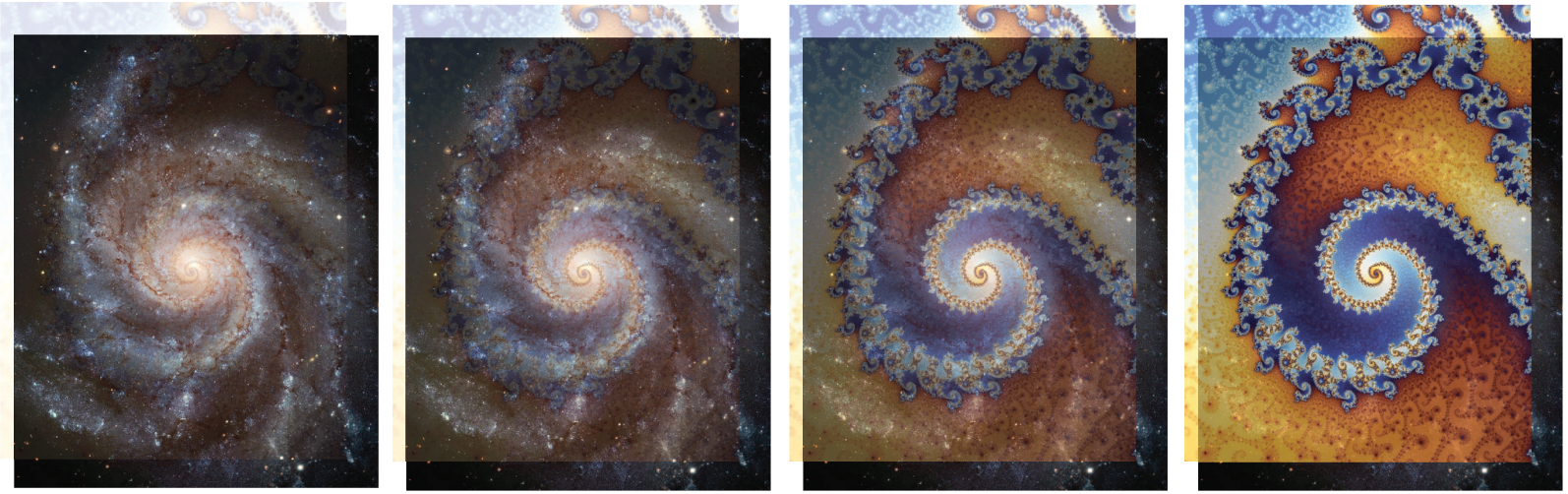
Gemstones and Galaxies

Adv. Combinatorics, University of Akron

January 11, 2014



*Nature image credits: NASA, Wikimedia and Google Earth. *"Nature" includes the Mandelbrot set.

Rough diamond.

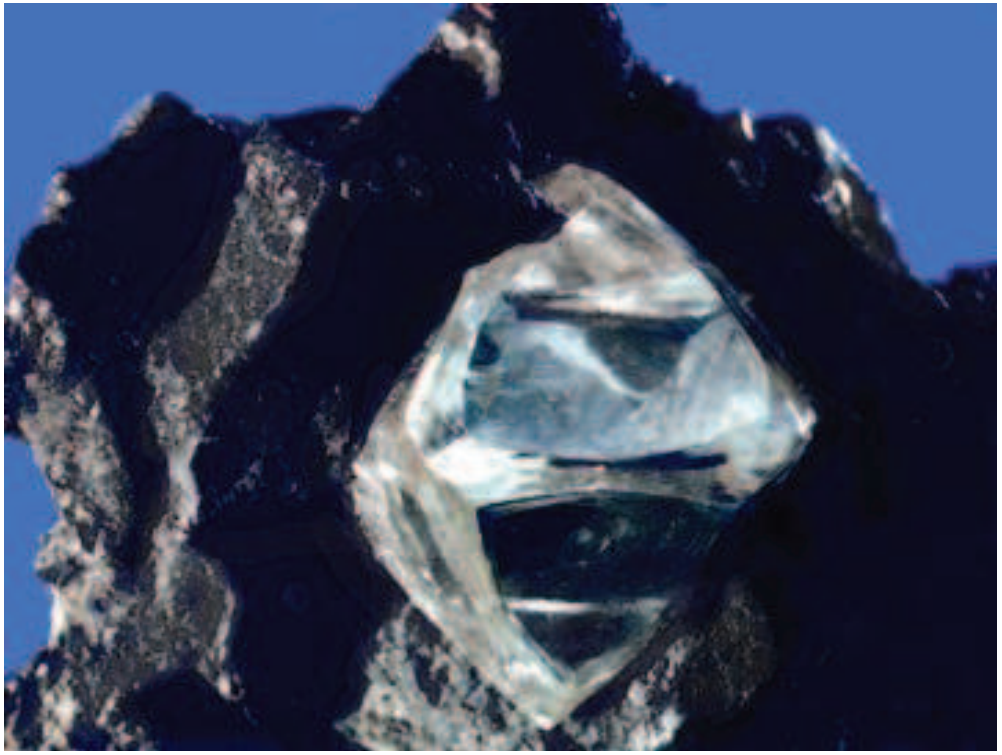


Image credits: Wikimedia.

Cut diamonds.

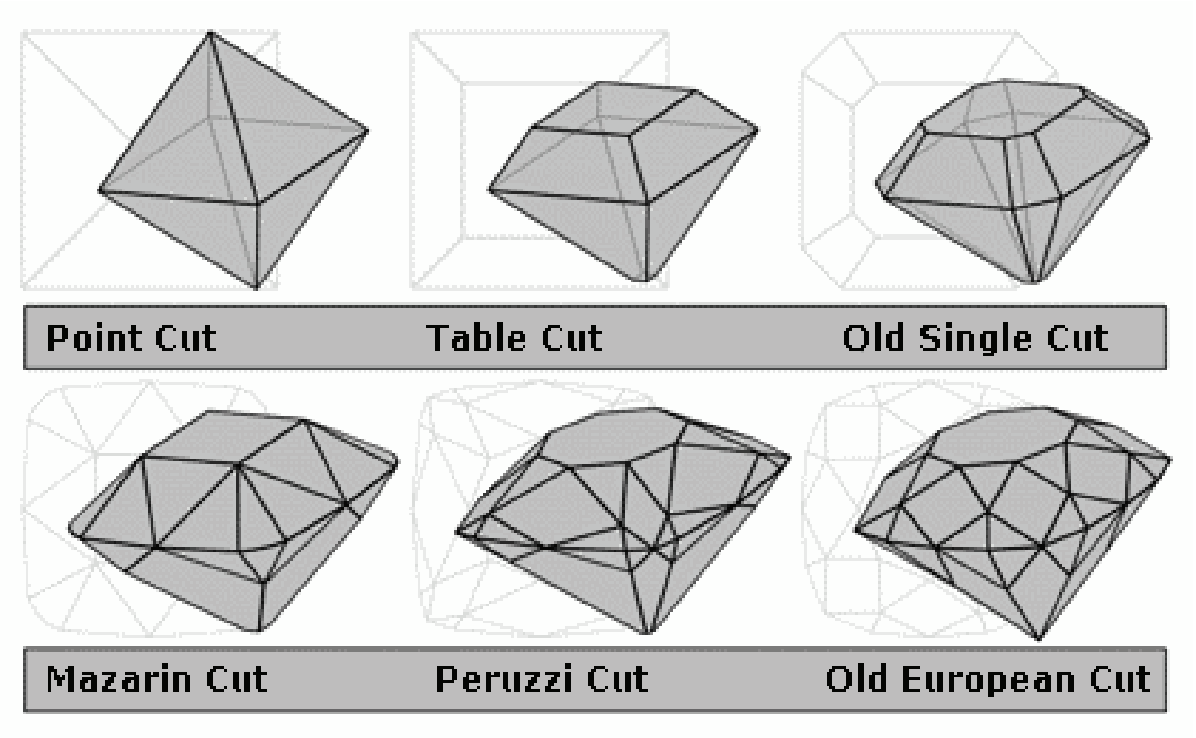


Image credits: Wikimedia.

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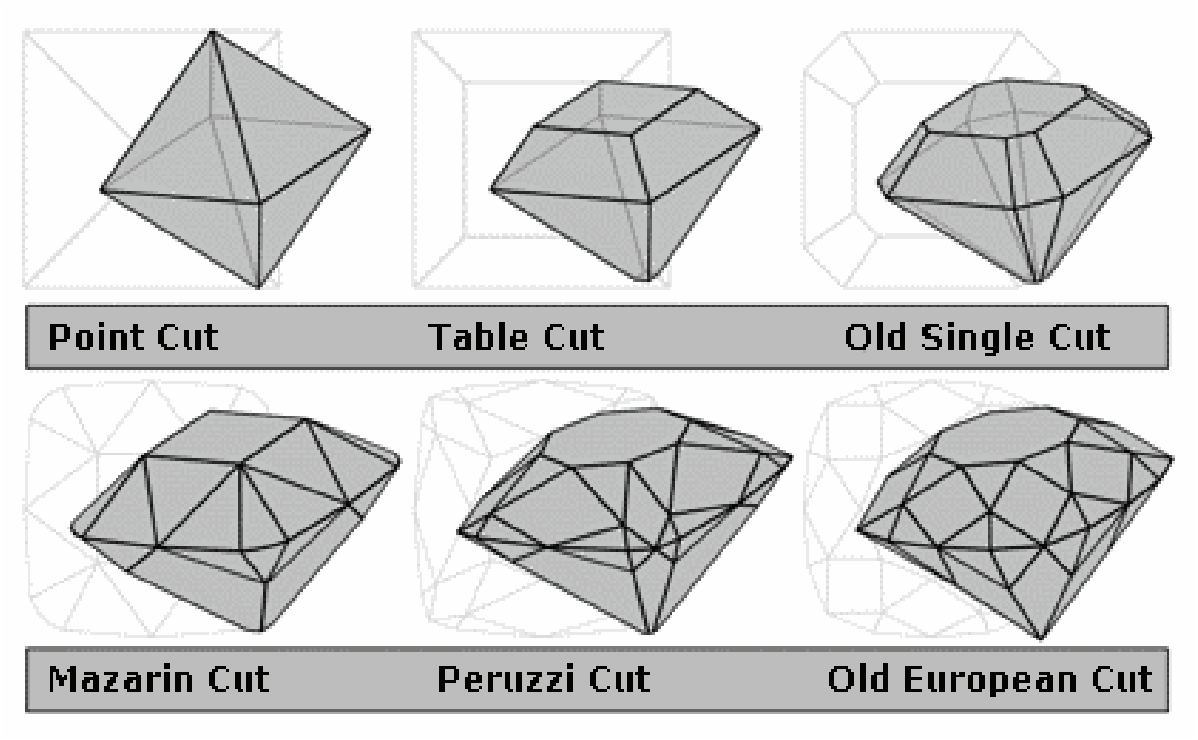
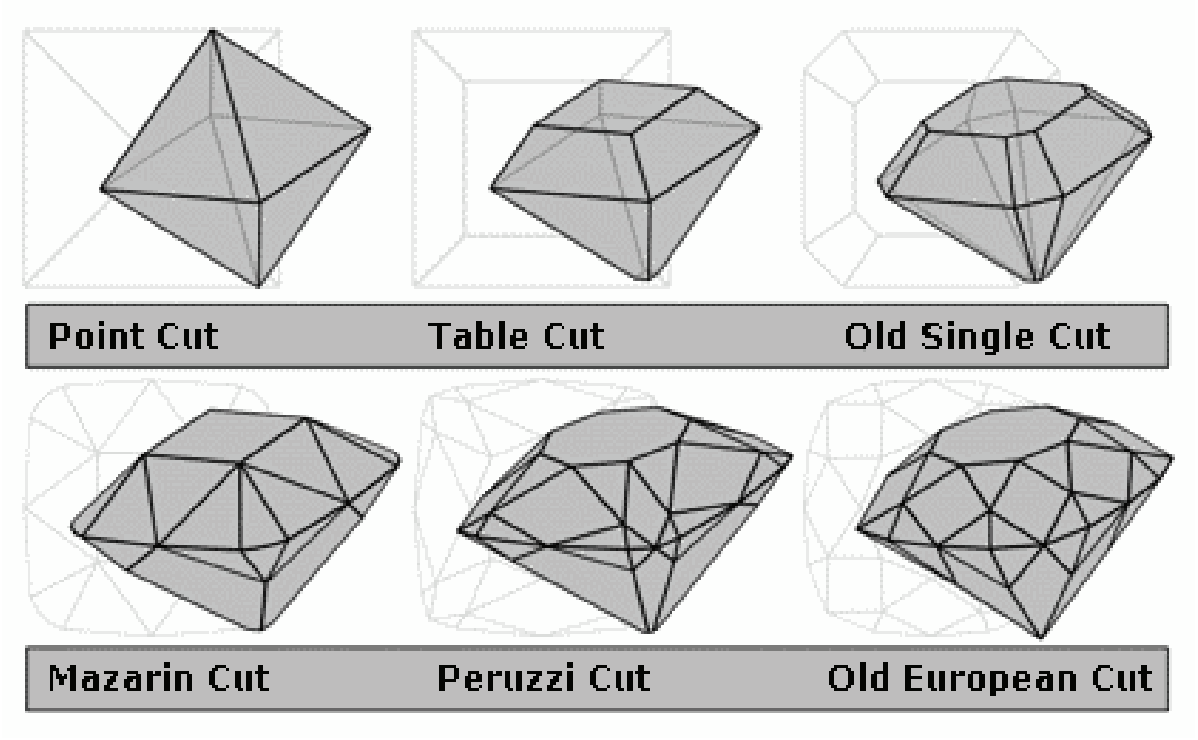


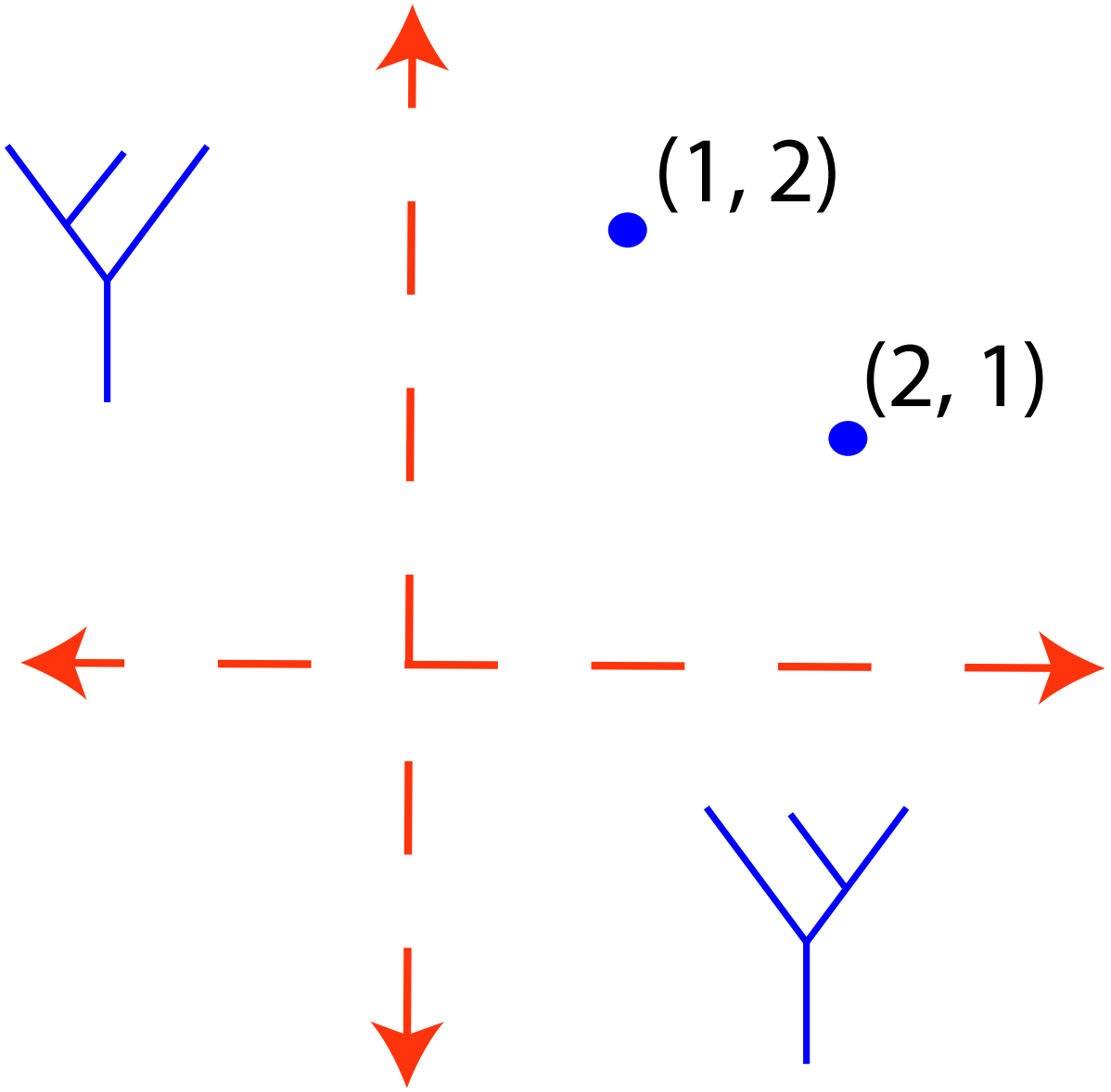
–Ten sides = 32300 types.

Cut diamonds.



–Ten sides = 32300 types.
–18 sides = 107854282197058 types (factor of a billion!)

A convex hull of binary trees.



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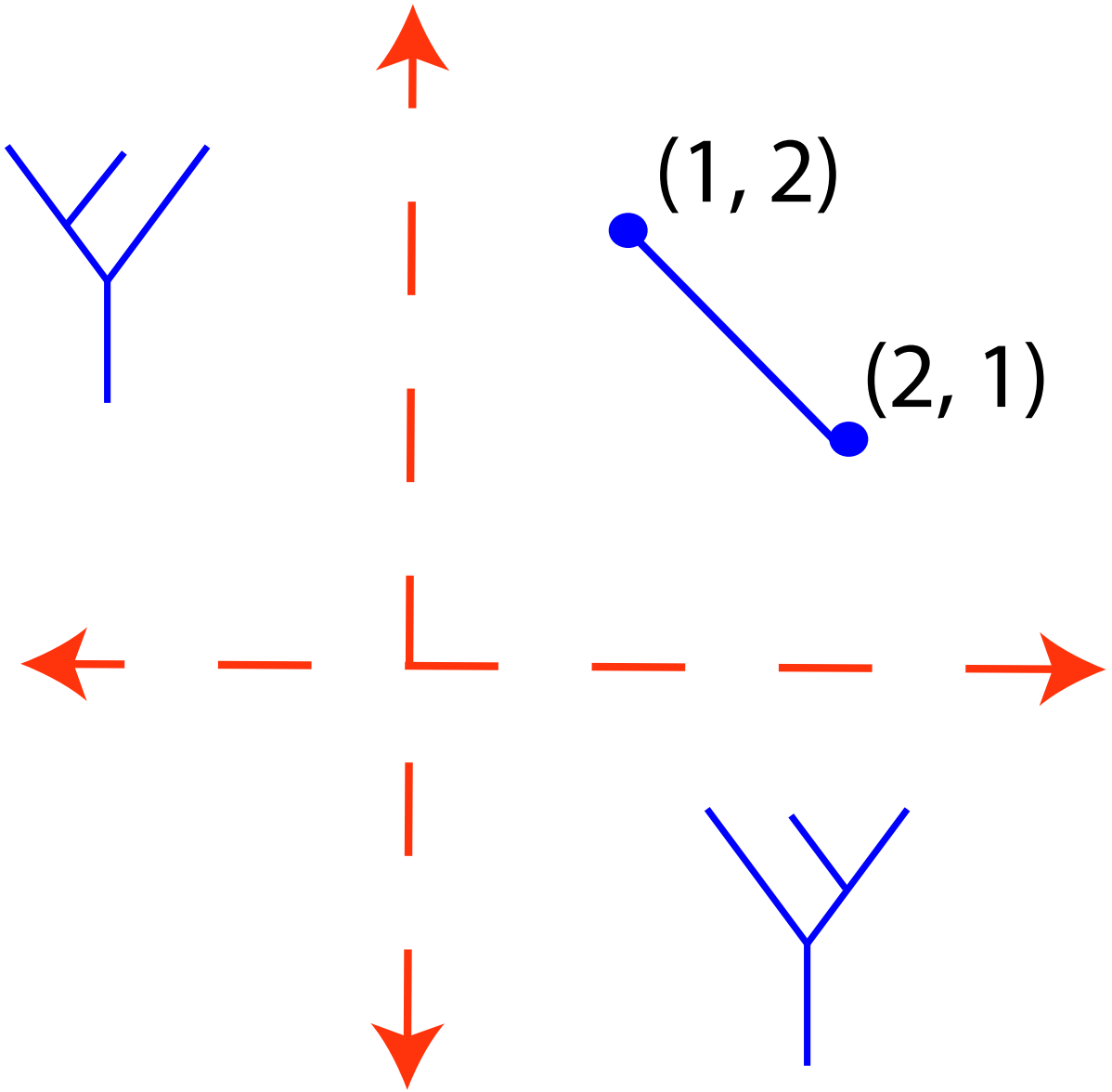

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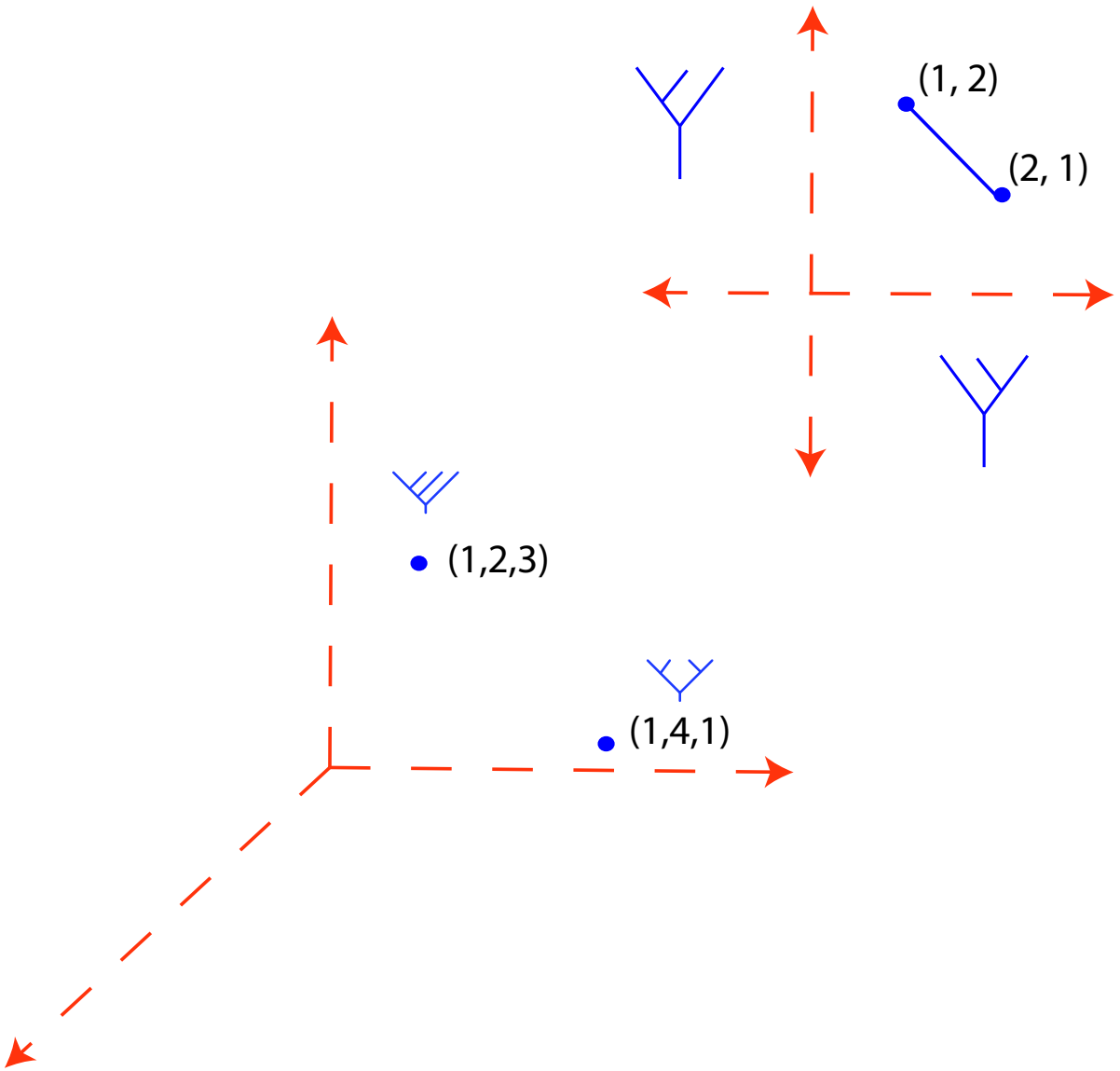

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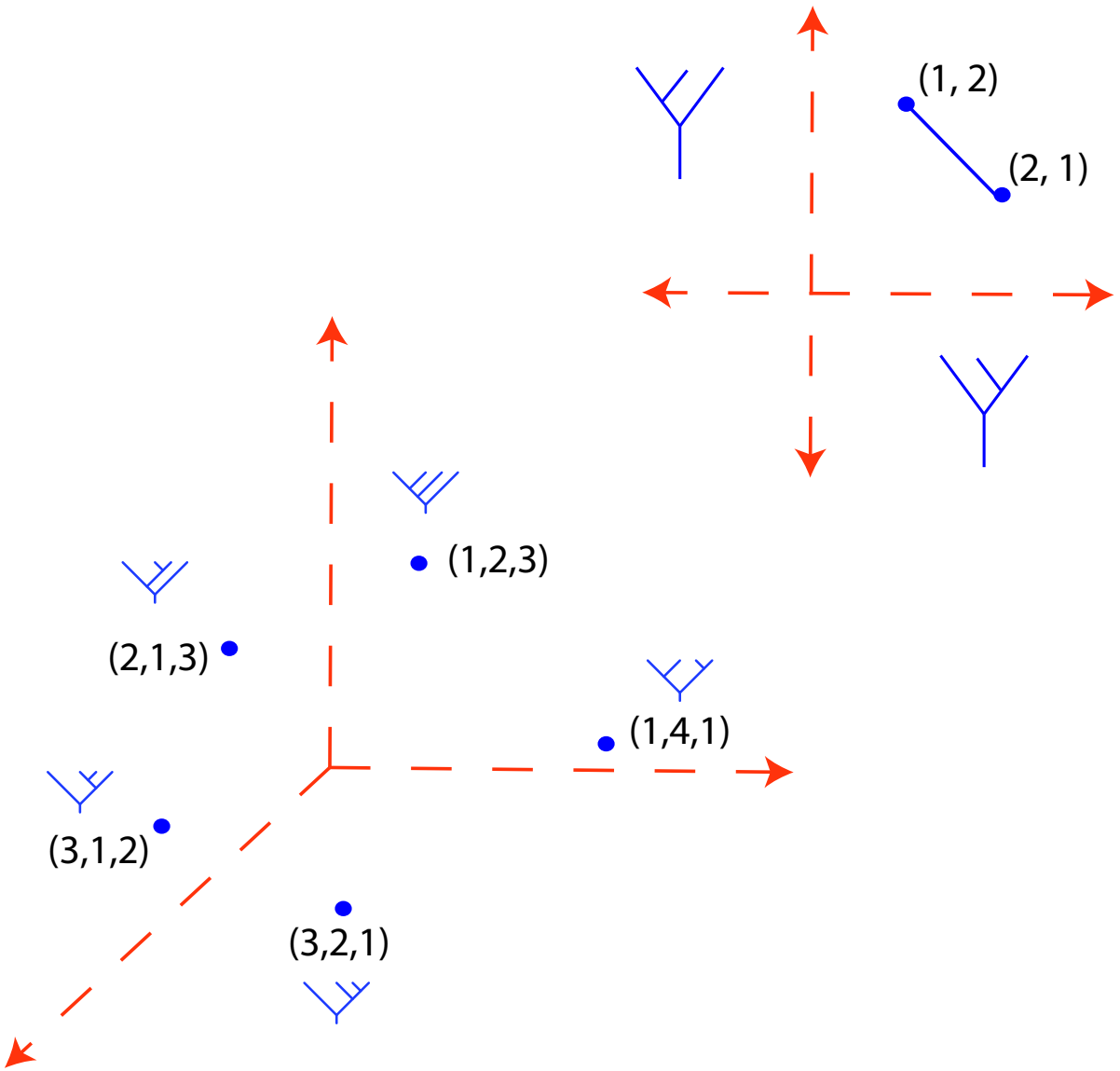

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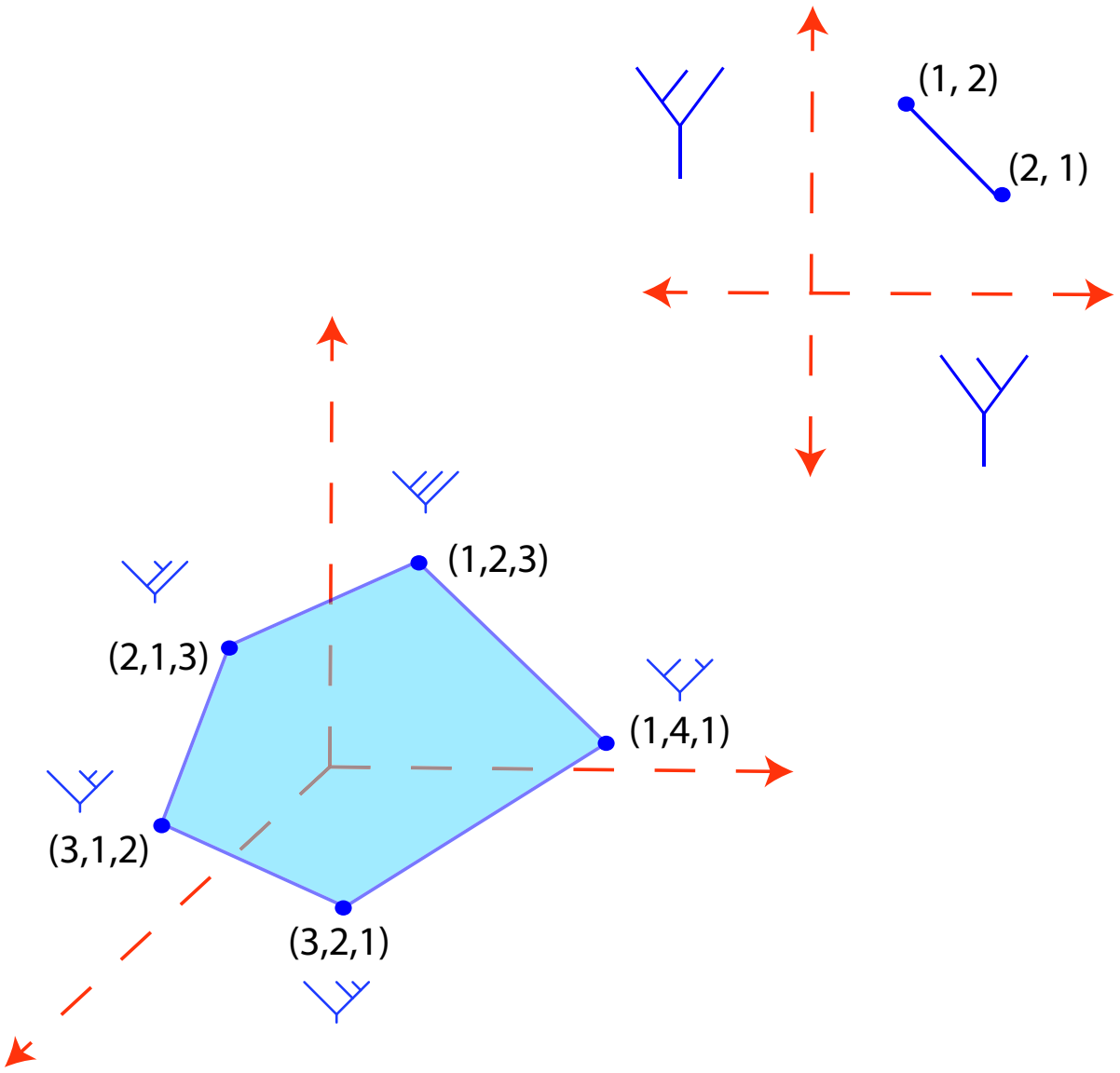

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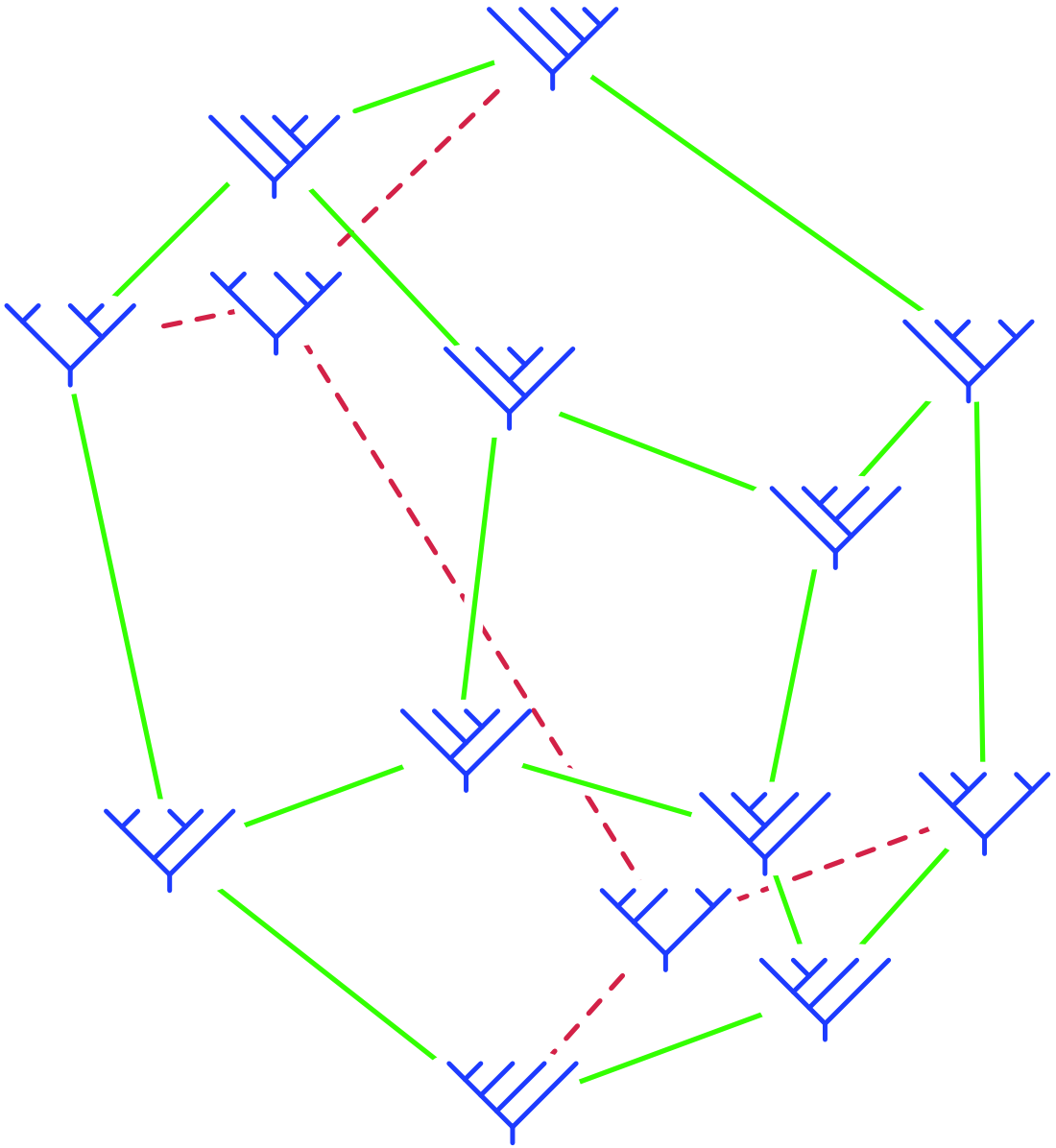
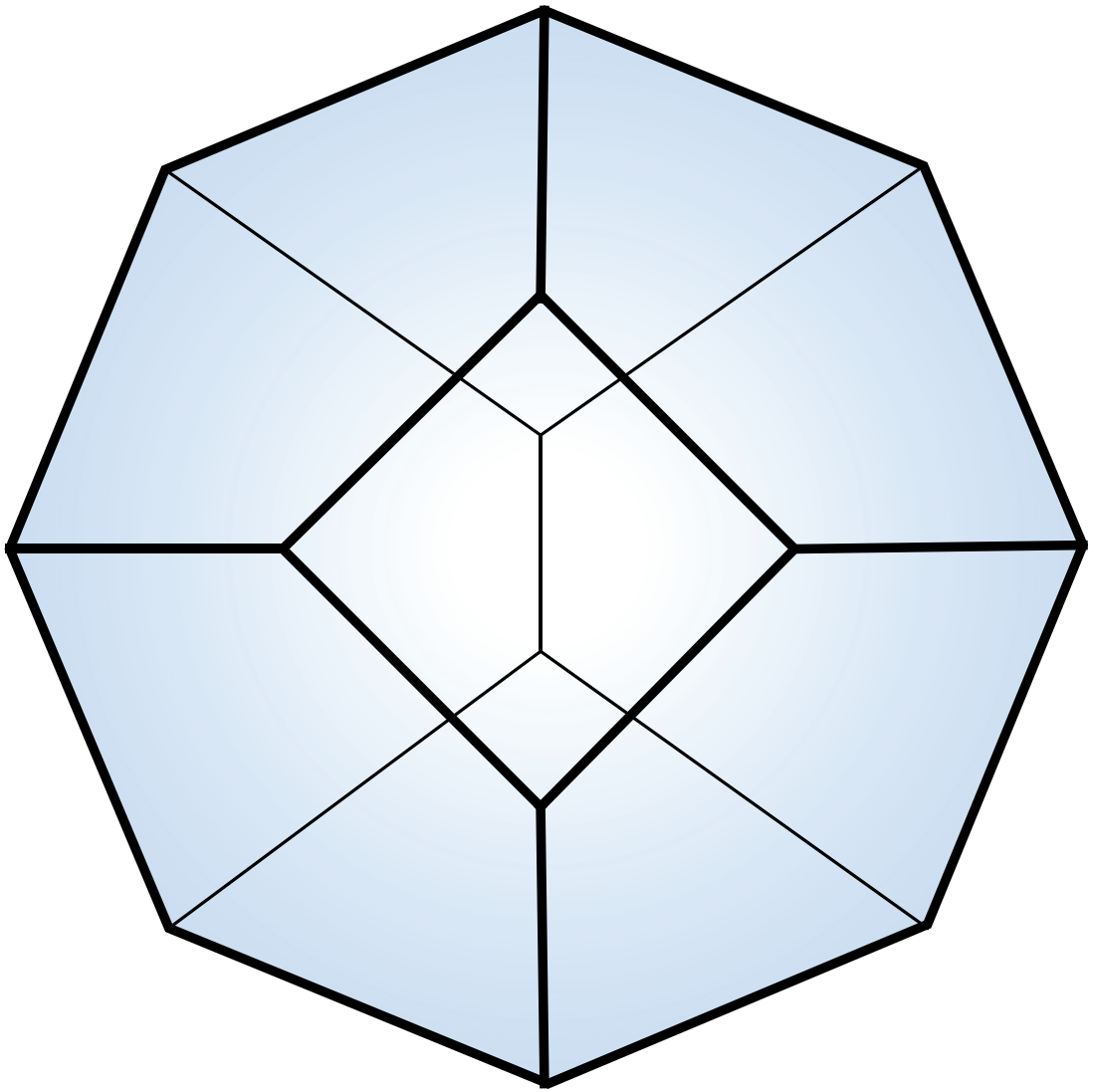
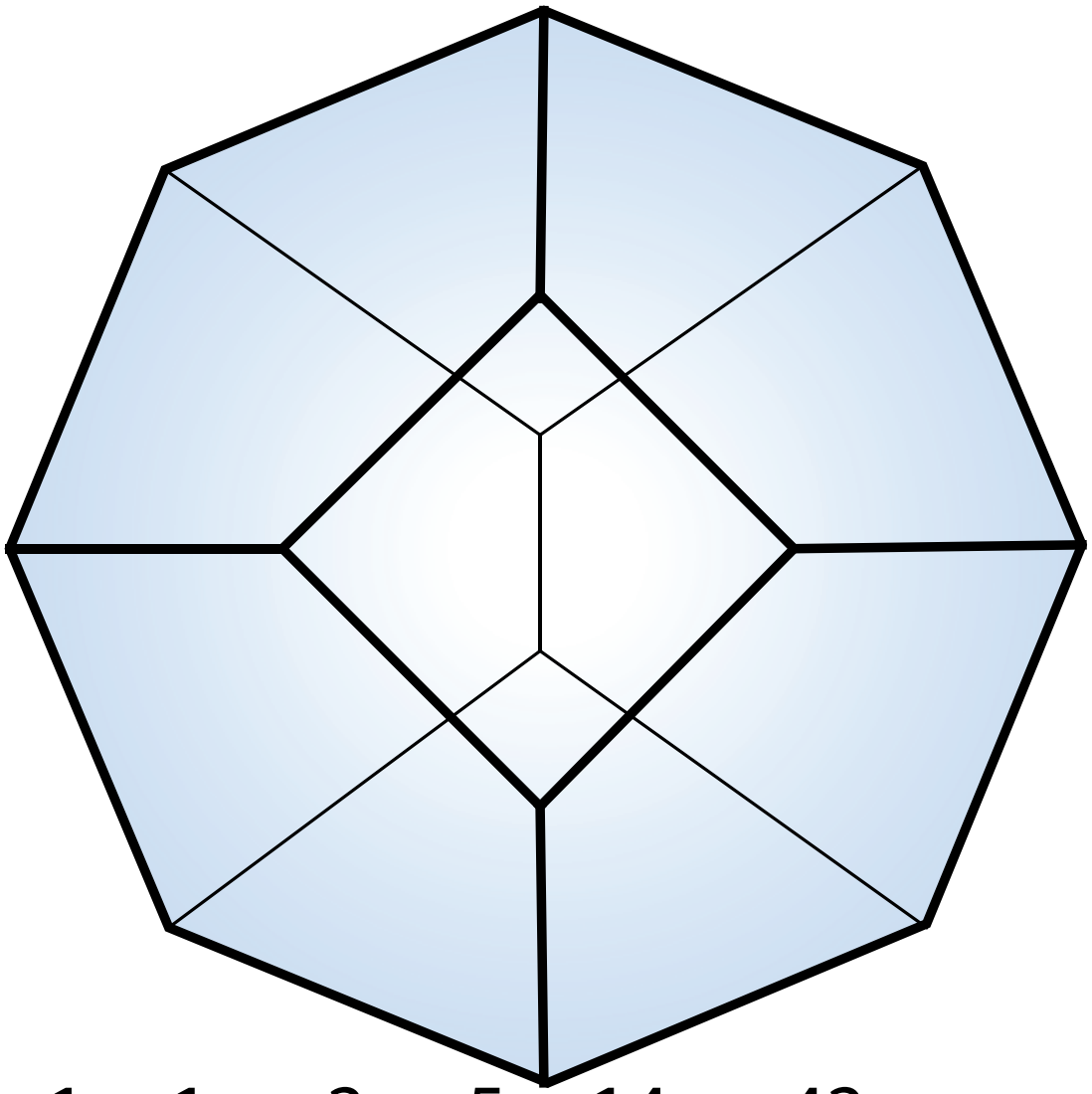


The associahedron.



The associahedra, and the Catalan numbers.



1 , 1 , 2 , 5 , 14 , 42 , ...

The Catalan numbers and the Mandelbrot sequences.

Definition

Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z.

$$z \mapsto z + z^2$$

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$$\mapsto z + z^2$$

$$\mapsto z + (z + z^2)^2$$

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$$= z + z^2 + 2z^3 + z^4$$

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$$= 1z + 1z^2 + 2z^3 + z^4$$

The Catalan numbers and the Mandelbrot sequences.

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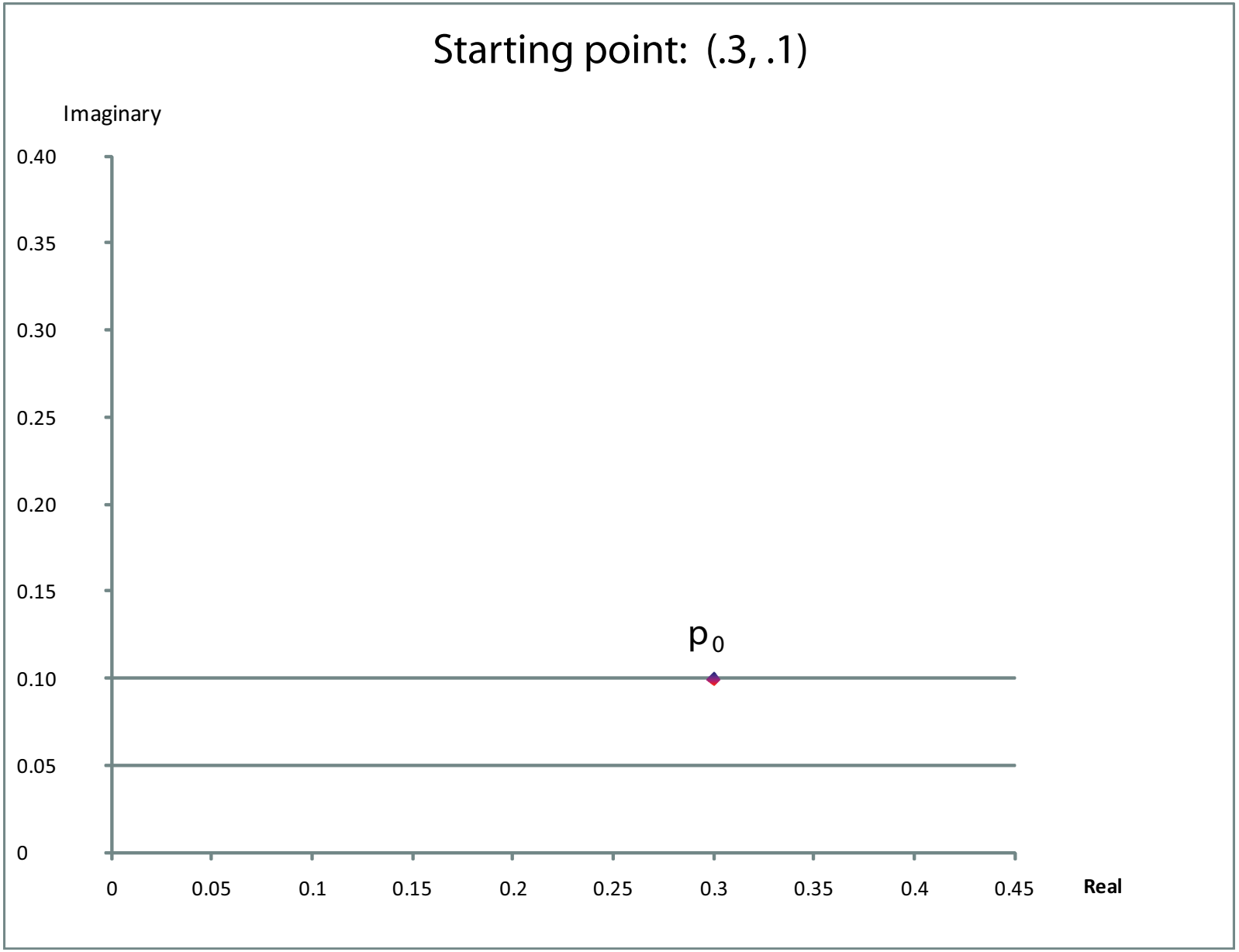
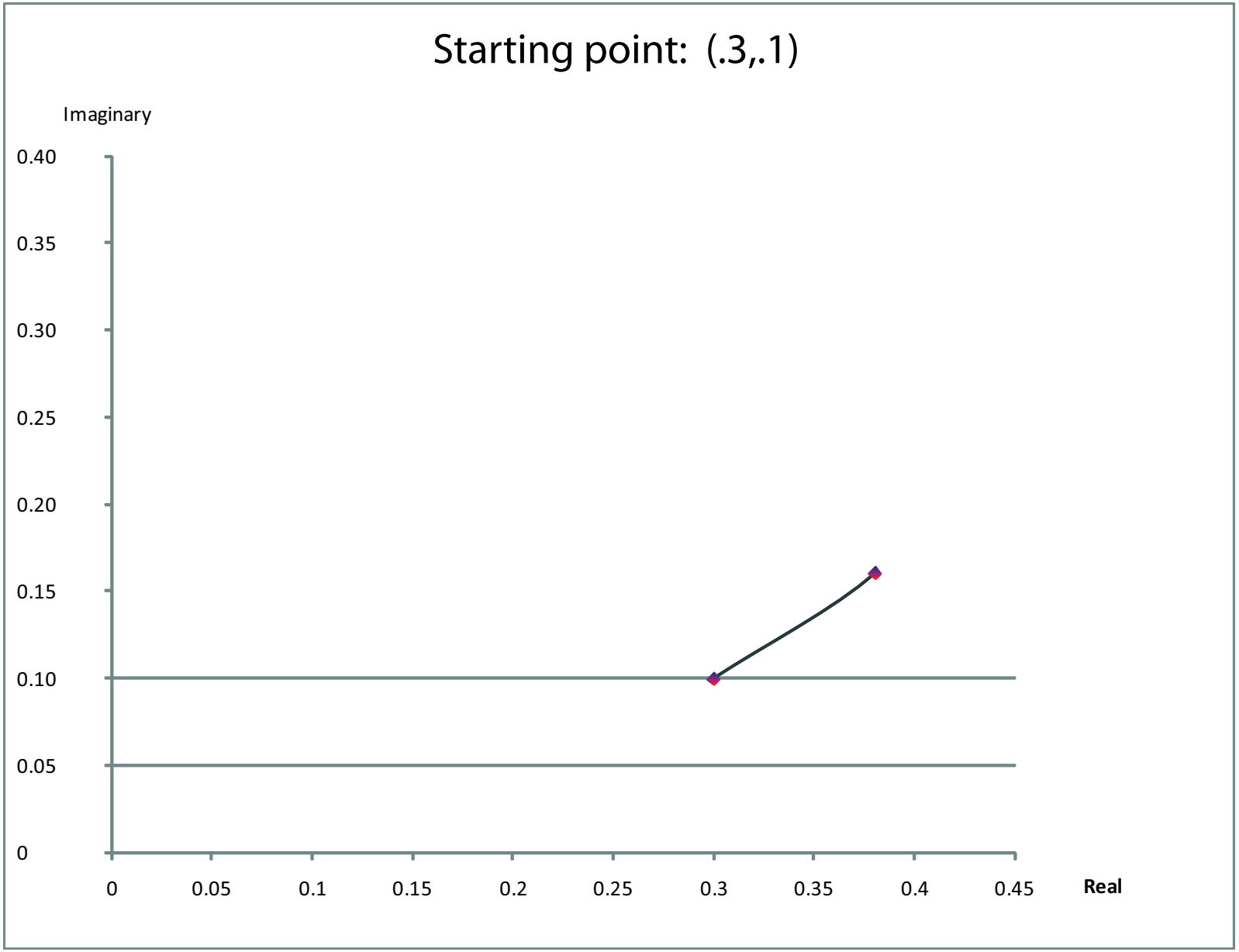
Mandelbrot sequences: Take a complex number z and add it to its square. Repeat, by squaring your result and adding the same original z.

$$
\begin{aligned}
z & \\
&\mapsto z + z^2 \\
&\qquad \mapsto z + (z + z^2)^2 \\
&\qquad\quad = 1z + 1z^2 + 2z^3 + z^4 \\
&\qquad\qquad \mapsto 1z + 1z^2 + 2z^3 + 5z^4 + 6z^5 + 6z^6 + 4z^7 + z^8 \\
&\qquad\qquad \vdots \\
&\qquad\qquad \mapsto 1z + 1z^2 + 2z^3 + 5z^4 + 14z^5 + 42z^6 + \dots
\end{aligned}
$$

Example Mandelbrot sequence: $z \mapsto z + z^2$



Mandelbrot $z \mapsto z + z^2$



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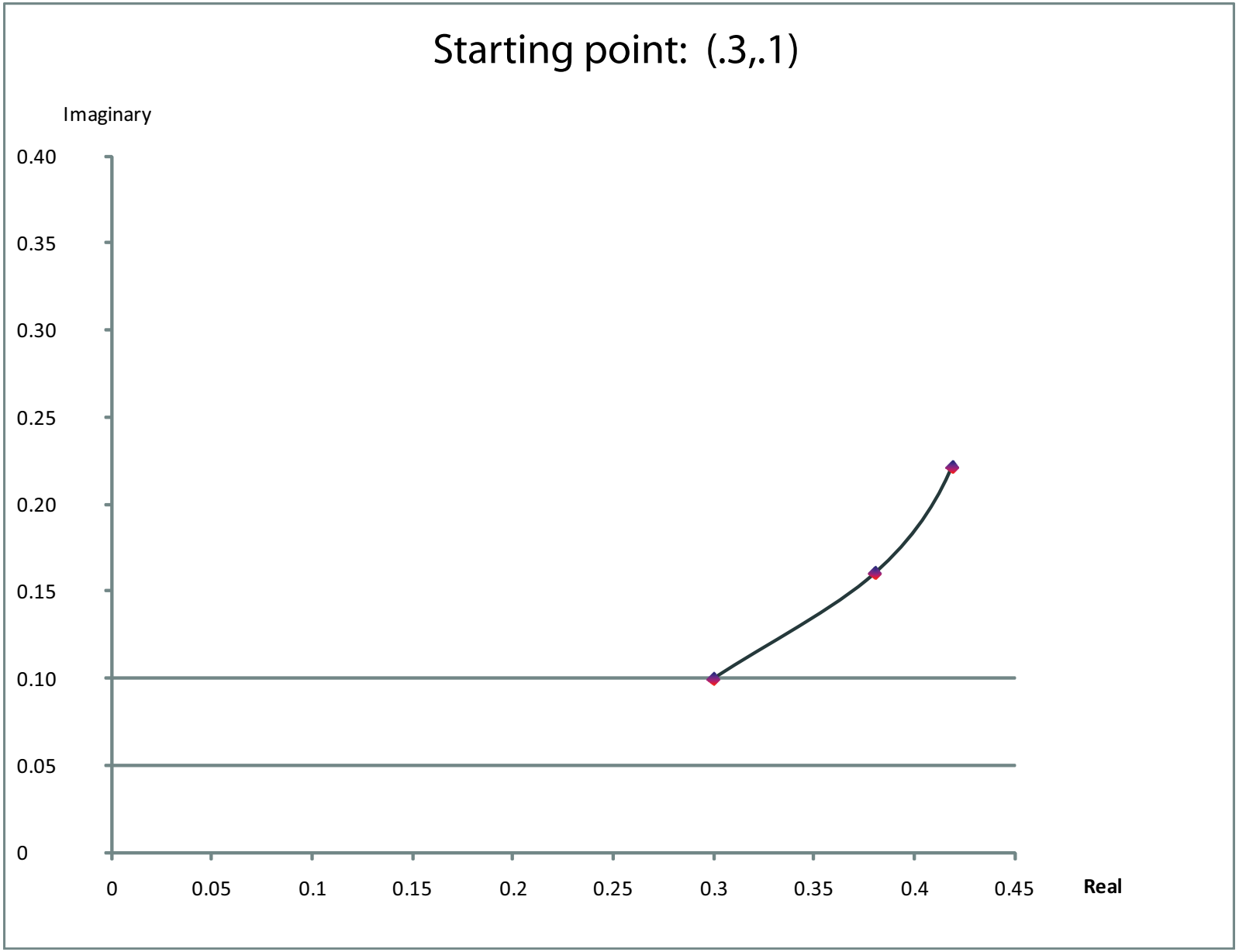
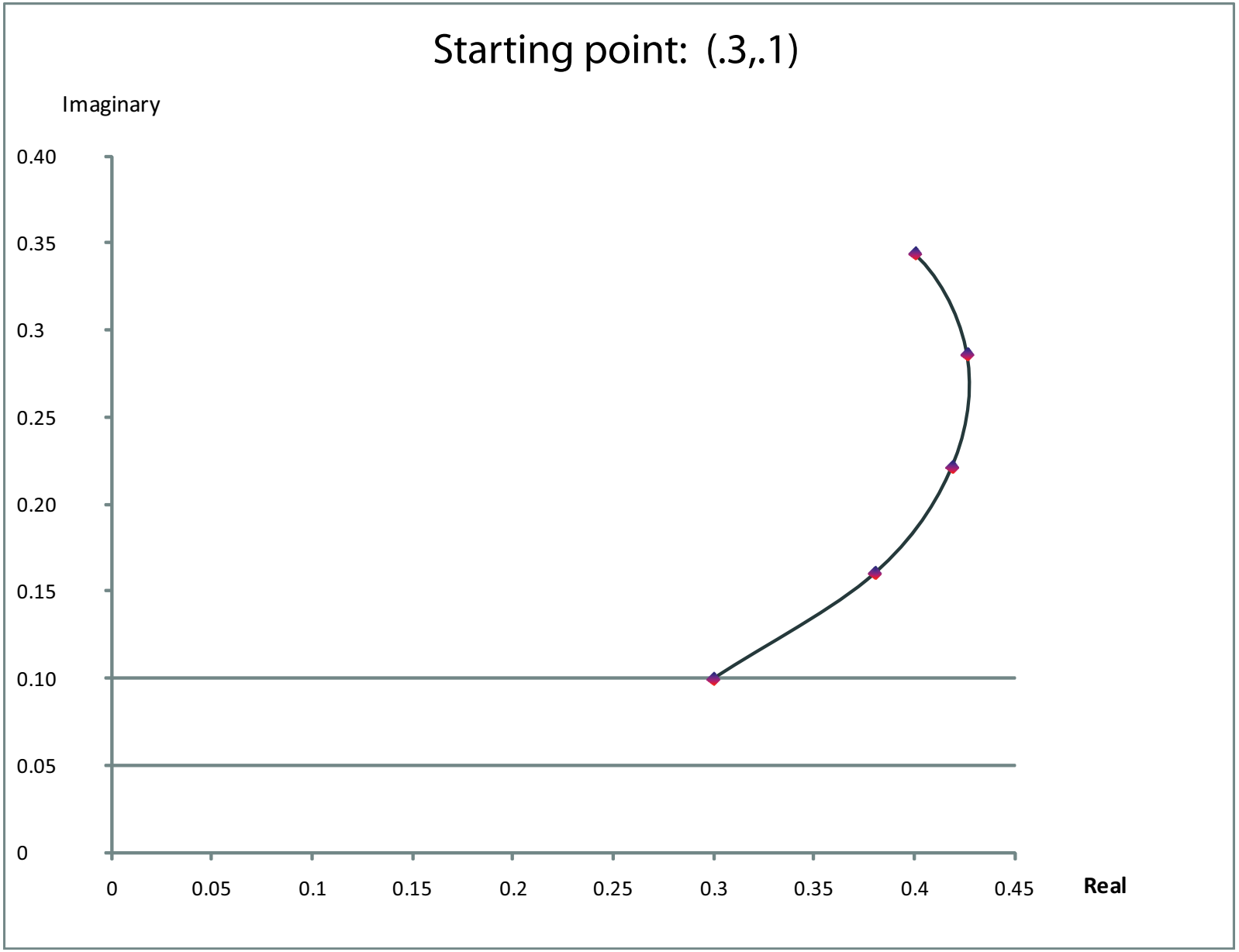
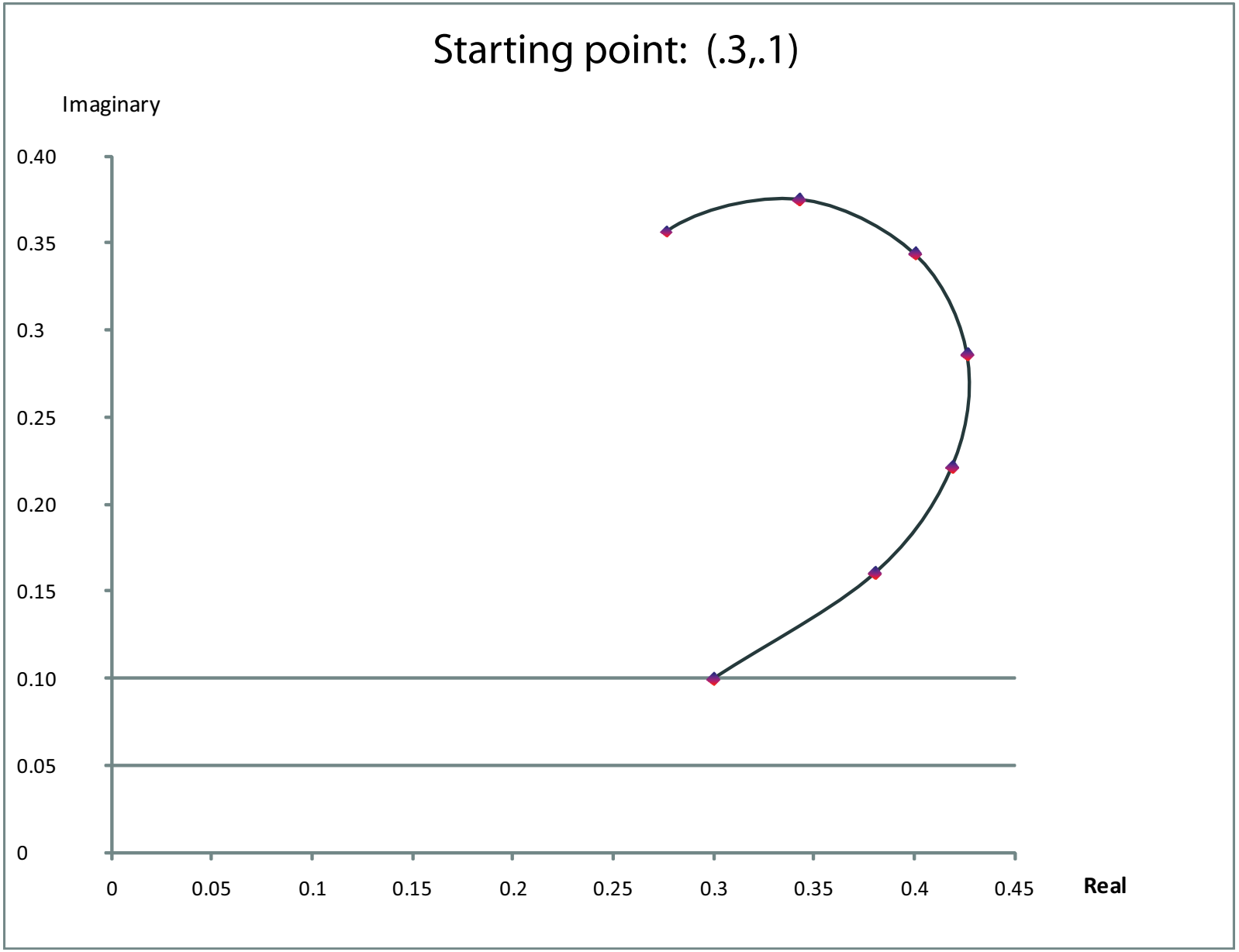


Mandelbrot $z \mapsto z + z^2$



Mandelbrot $z \mapsto z + z^2$



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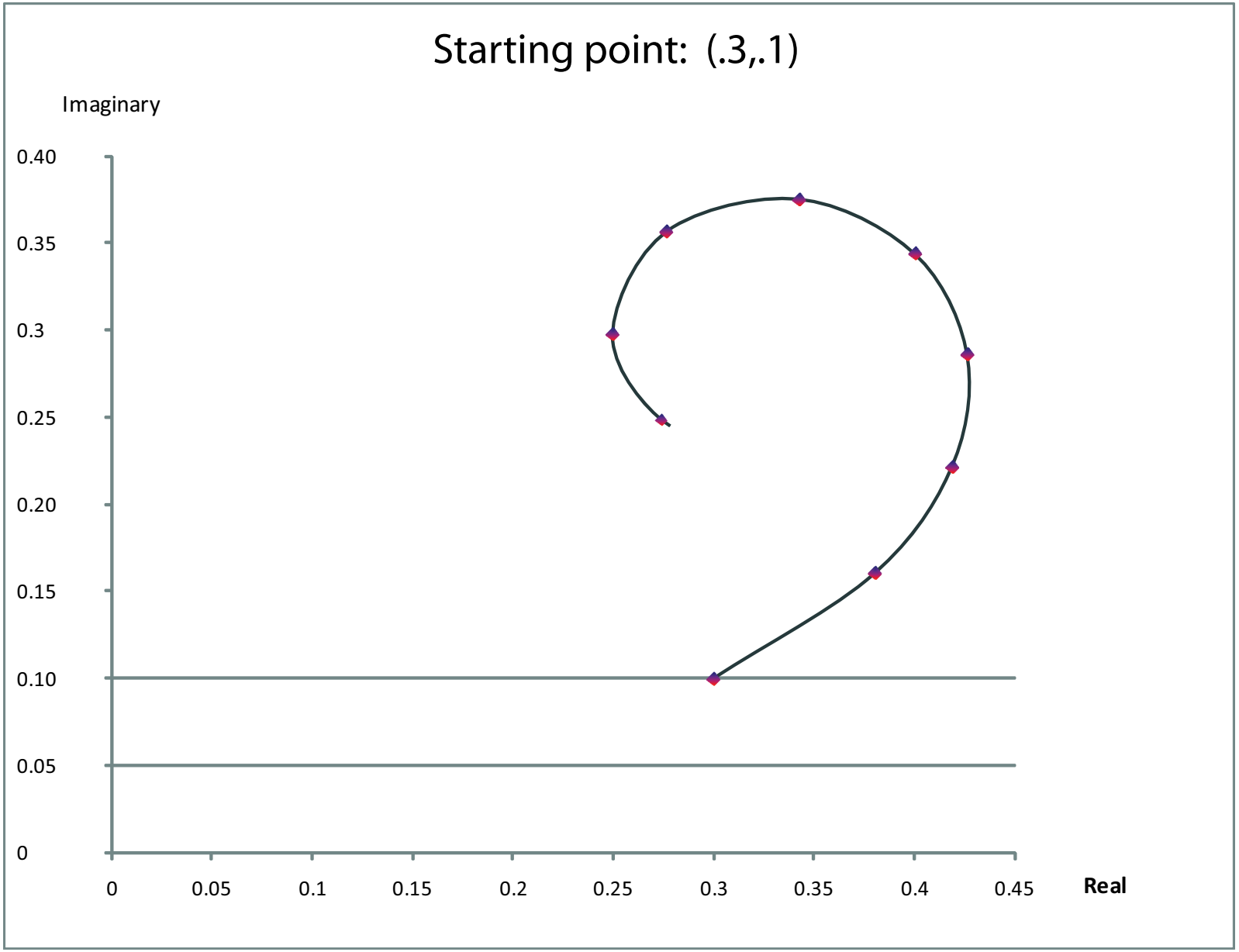

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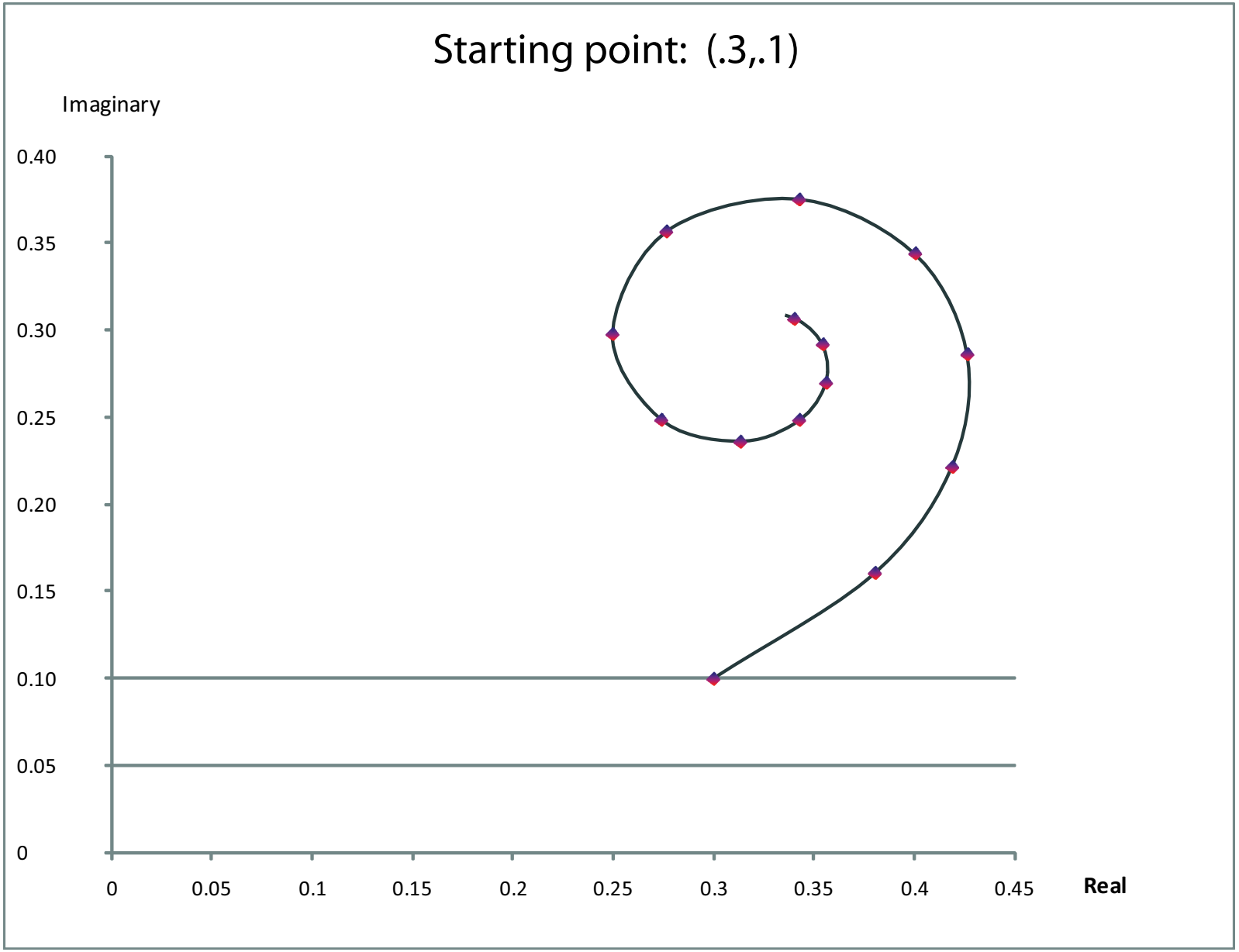

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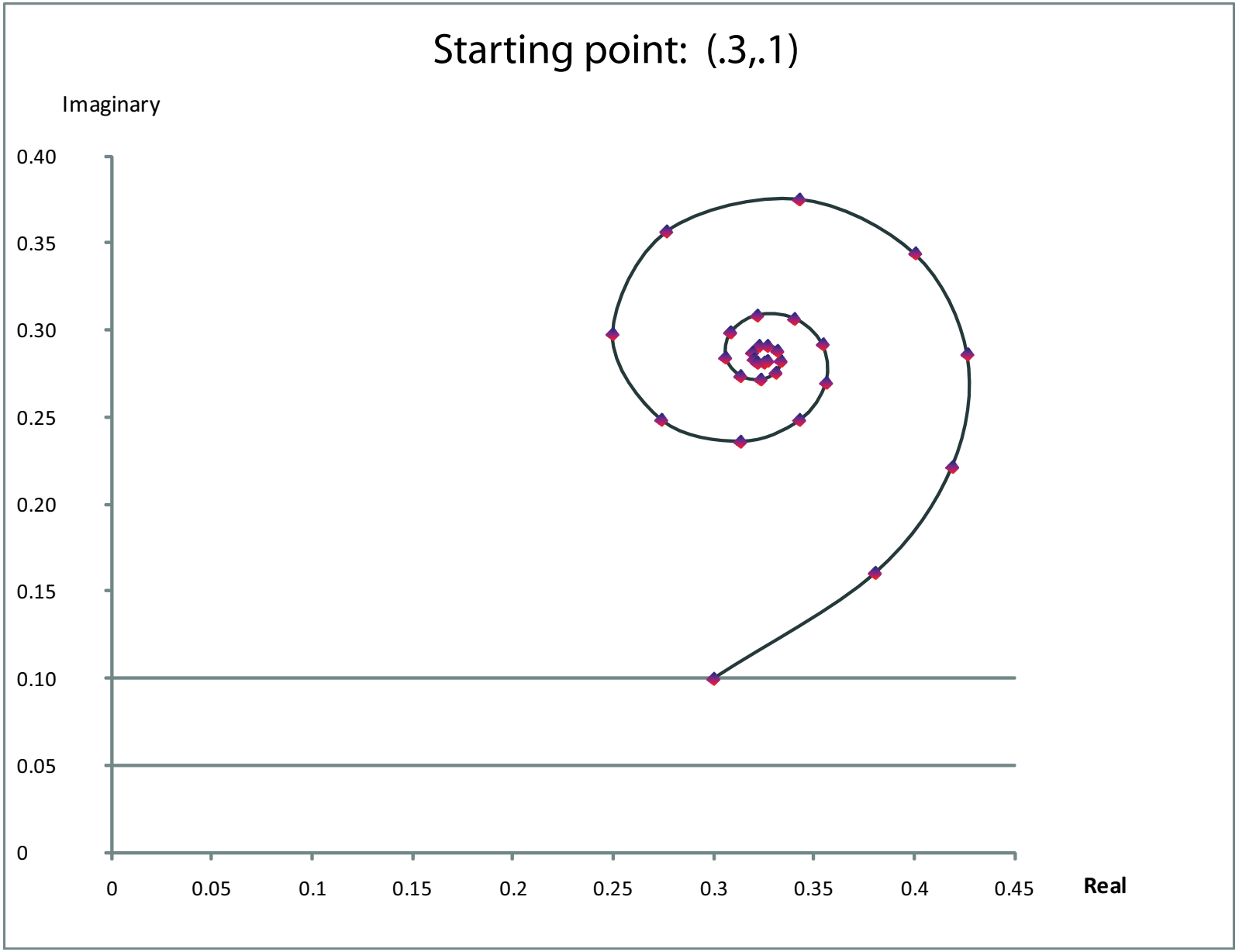
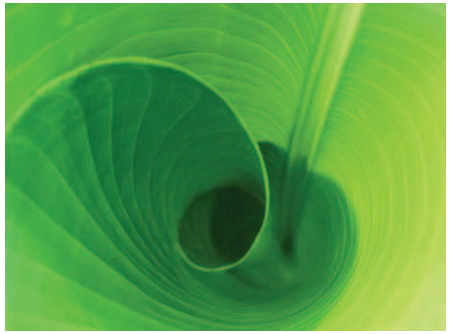
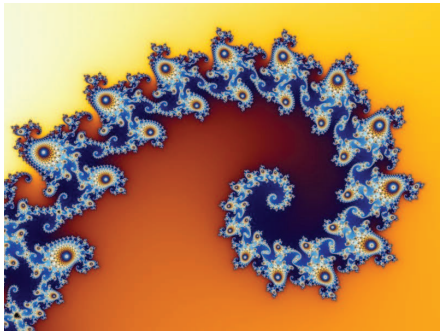


Logarithmic spiral.

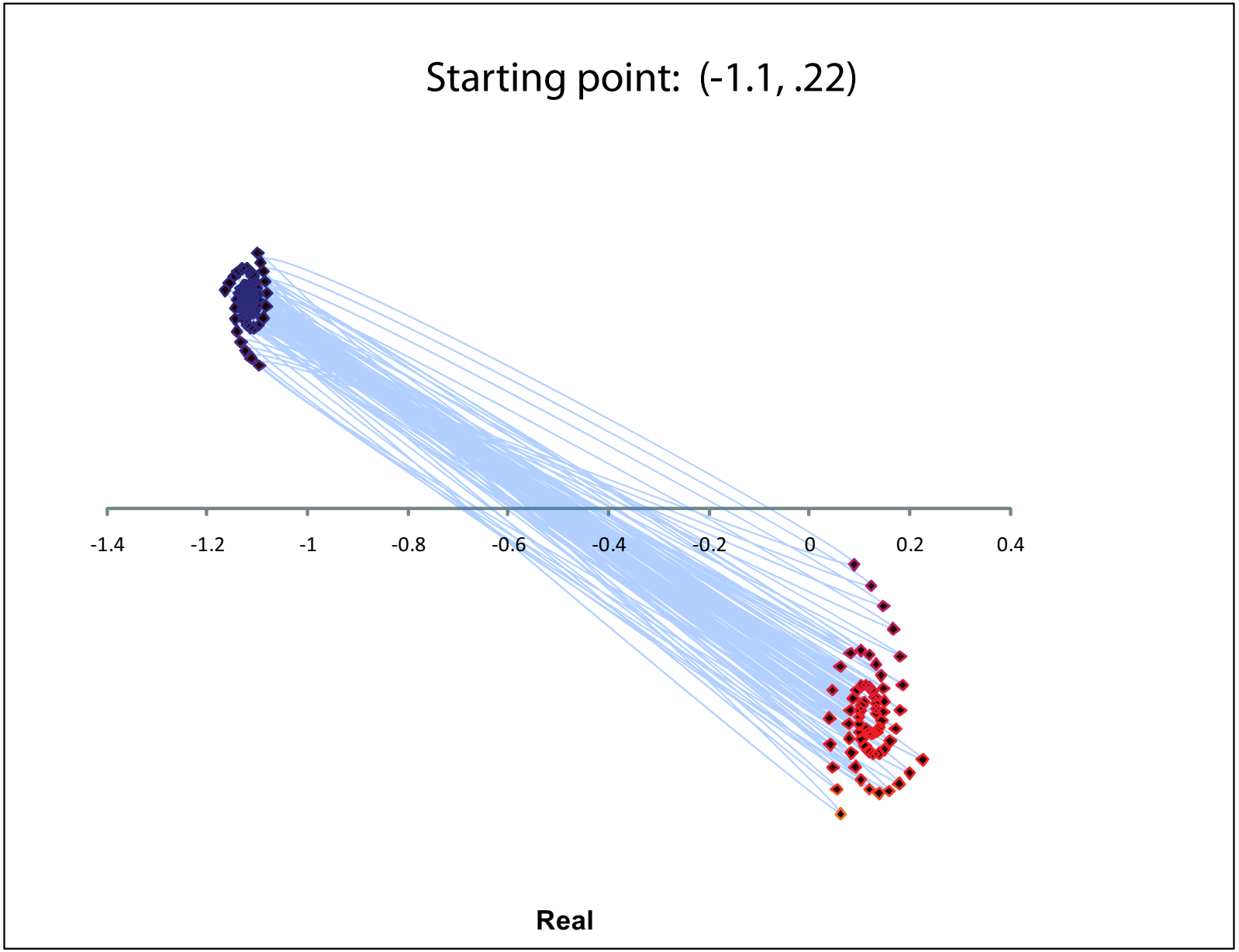
$$r = e^{\theta}.$$

Imag

credits: McSweeney's (first two) Wikimedia (last two).

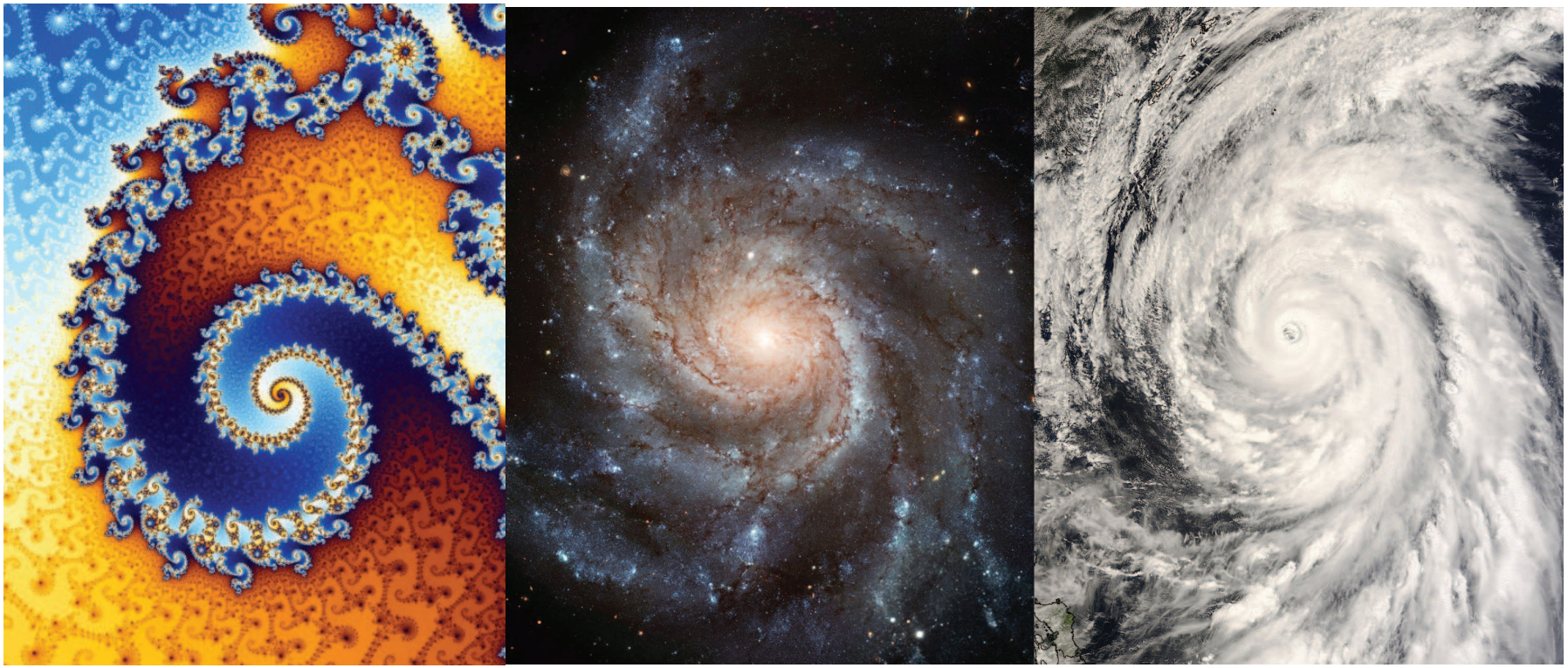
Mandelbrot $z \mapsto z + z^2$



Logarithmic spiral.

$$\ln r = \theta.$$

Image credits: Wikimedia (first), NASA (second two).



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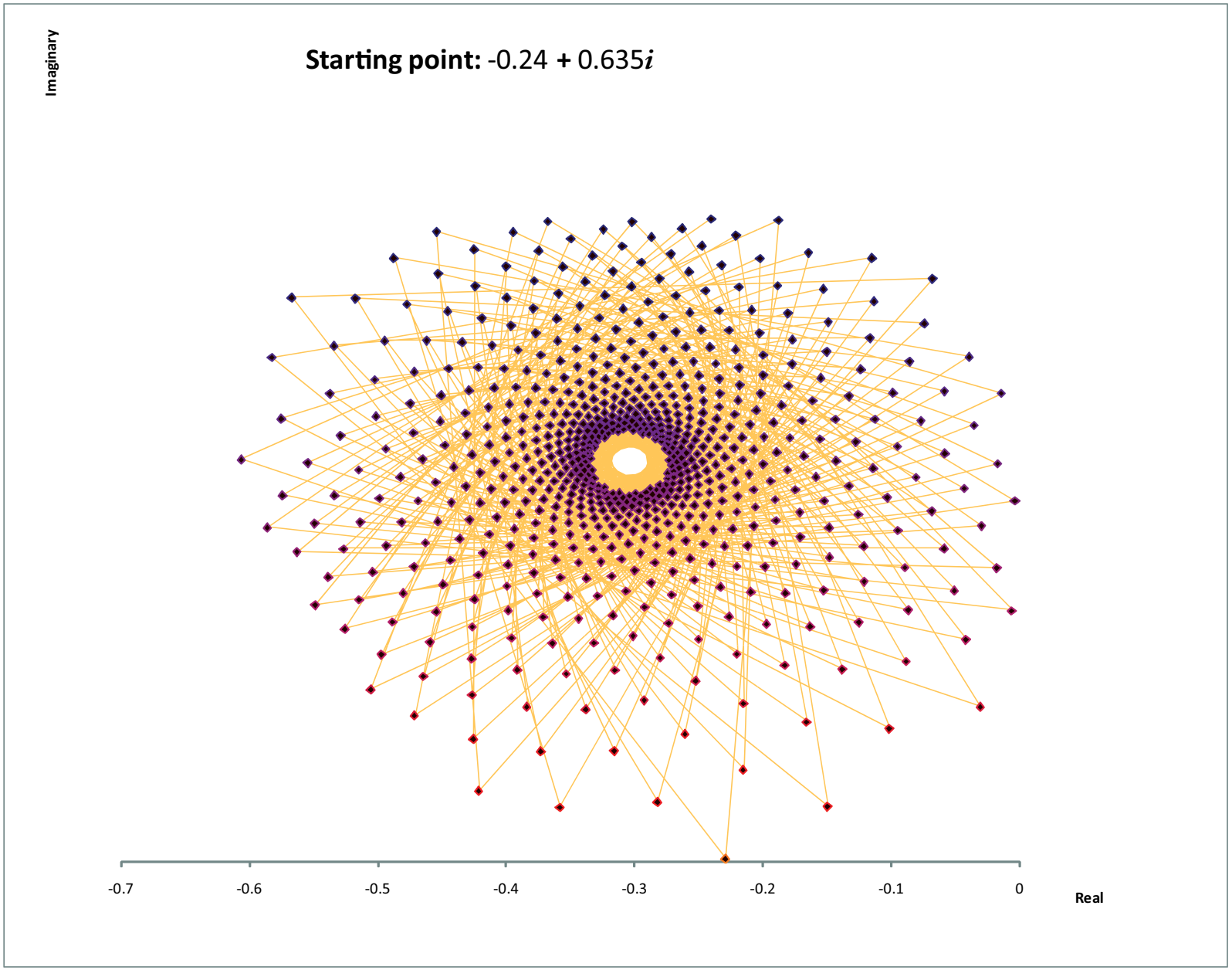

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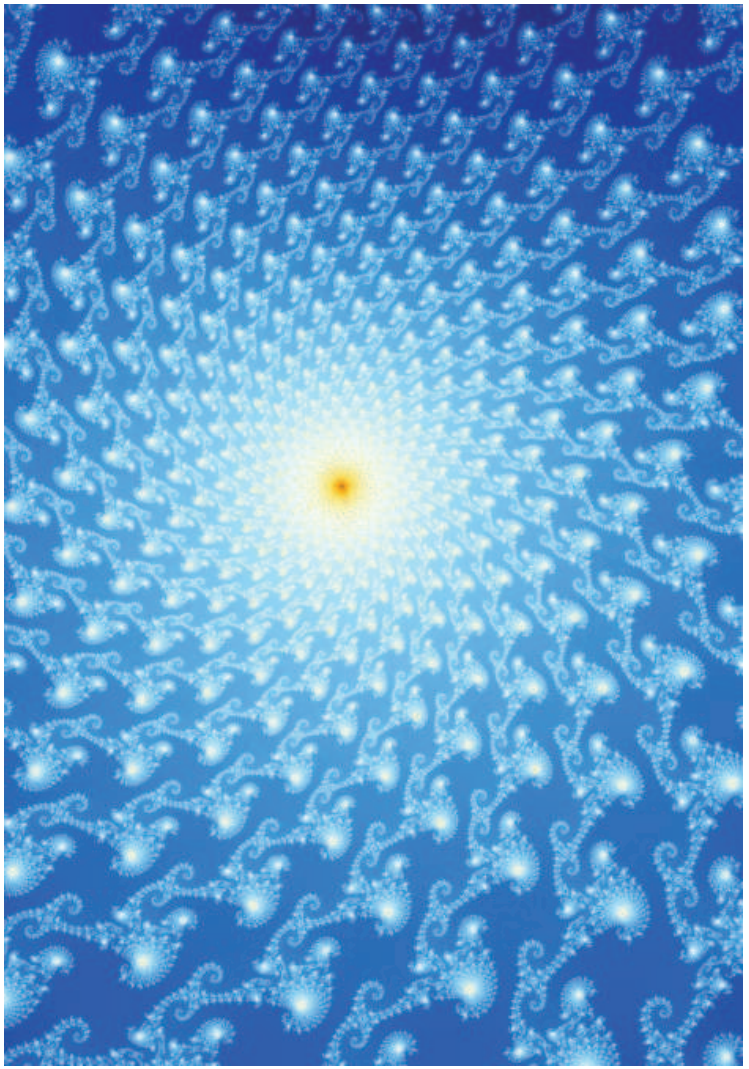

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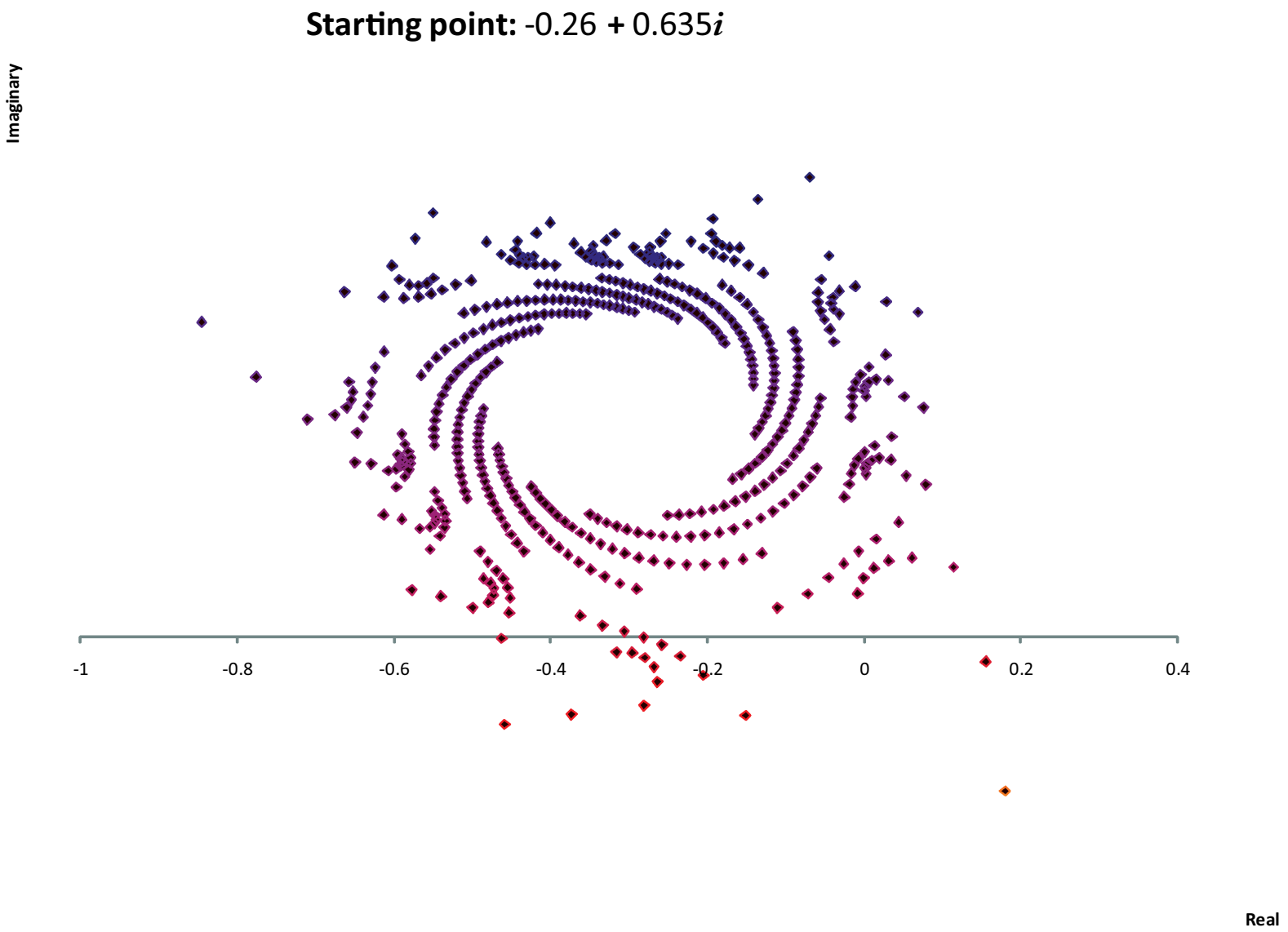
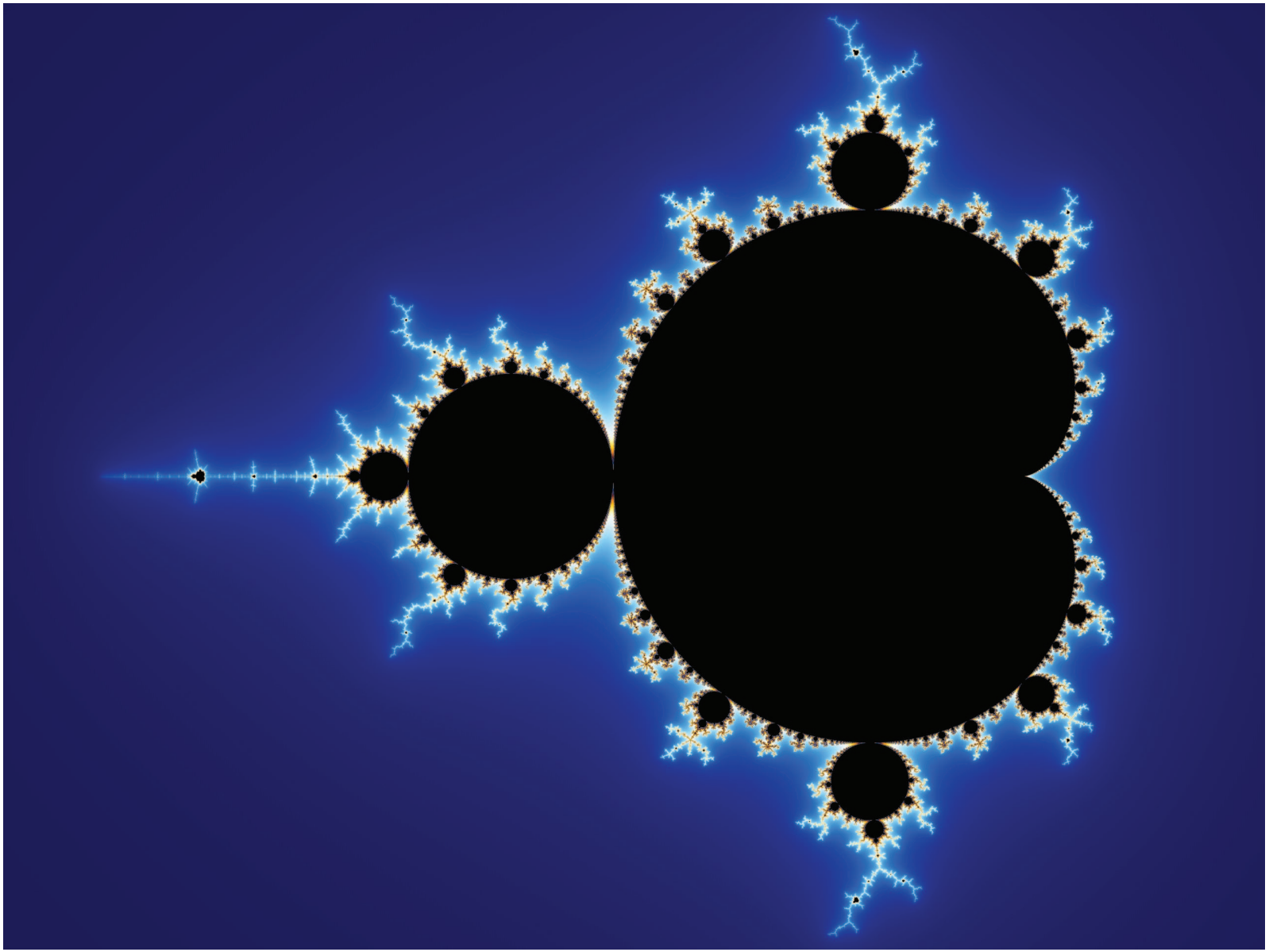


Image credits (all following Mandelbrot slides):
Wikimedia, Wolfgang Beyer.
http://commons.wikimedia.org/wiki/User:Wolfgangbeyer

Mandelbrot set.



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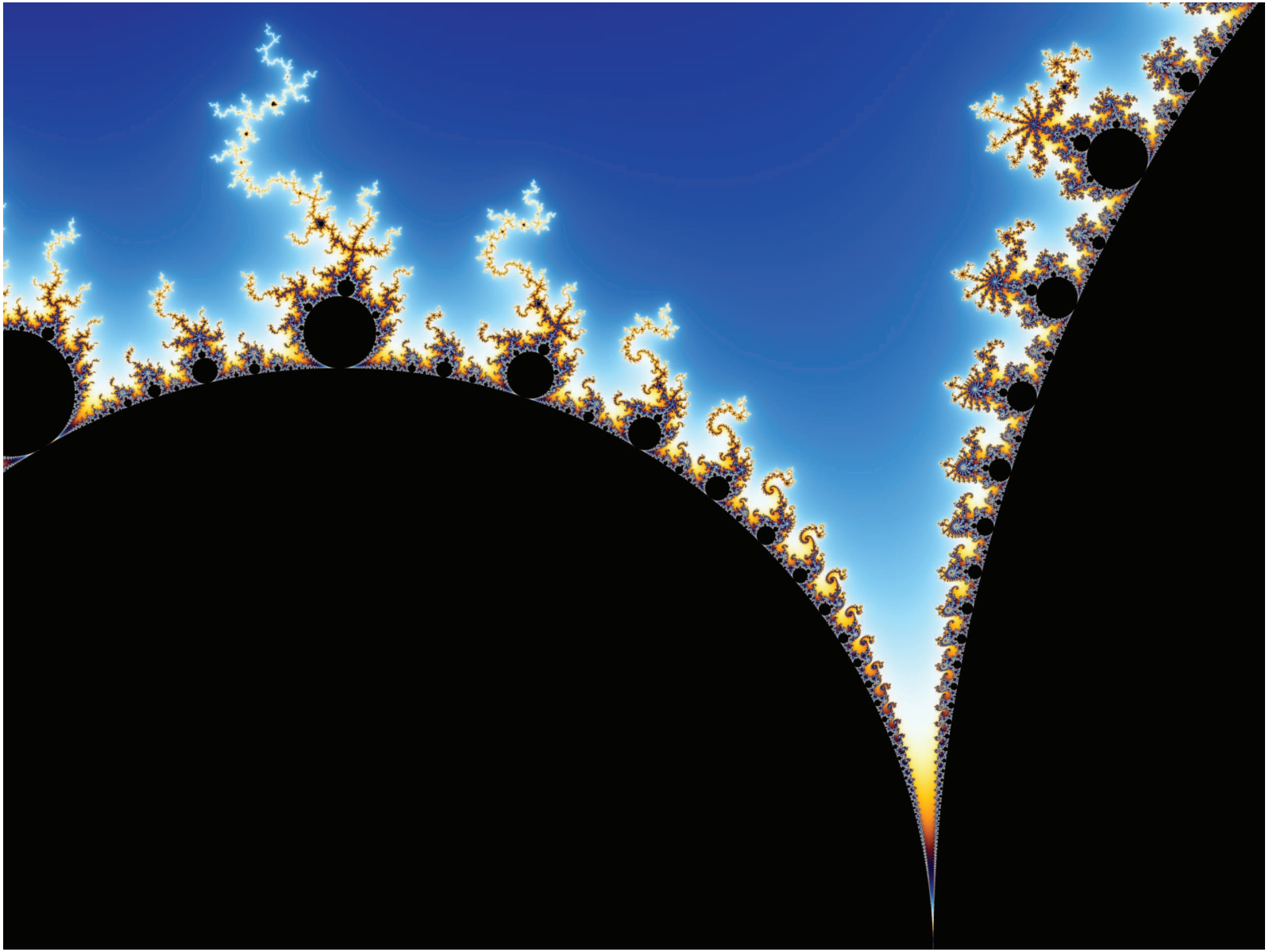
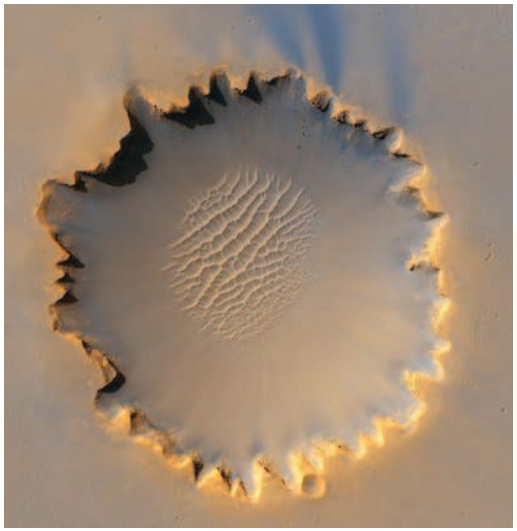


Fractal

Victoria crater.



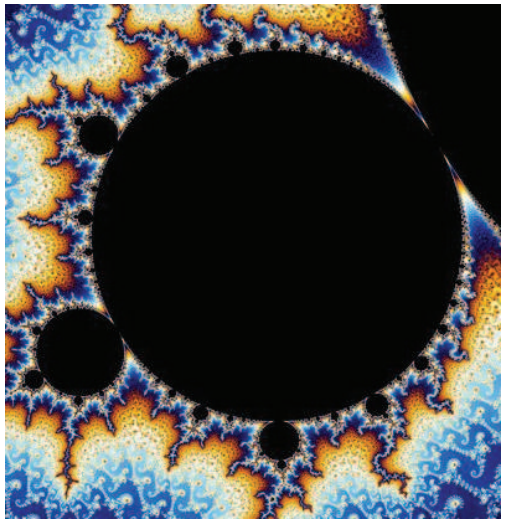
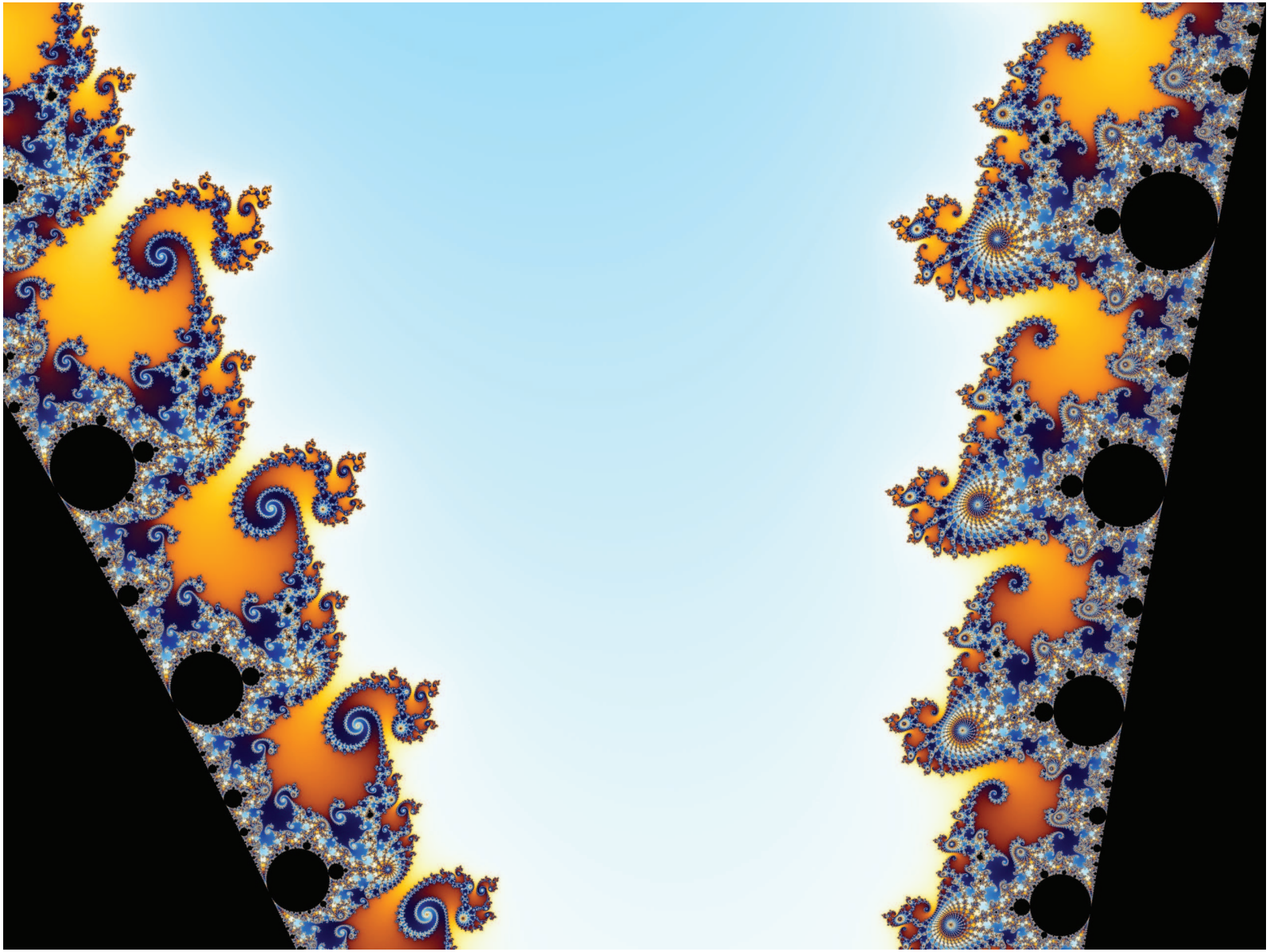


Image credit for Victoria crater: NASA.

Mandelbrot set.



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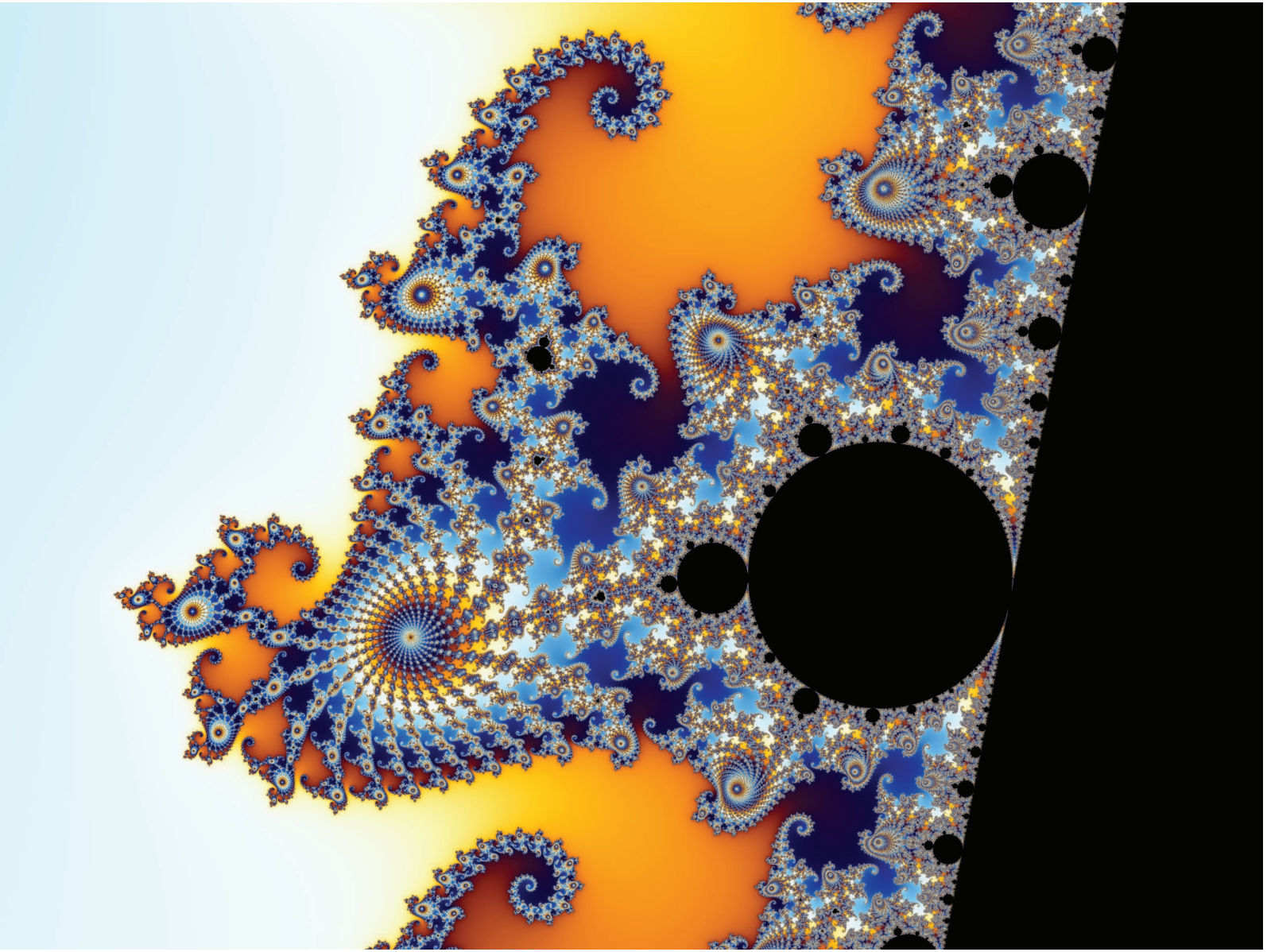

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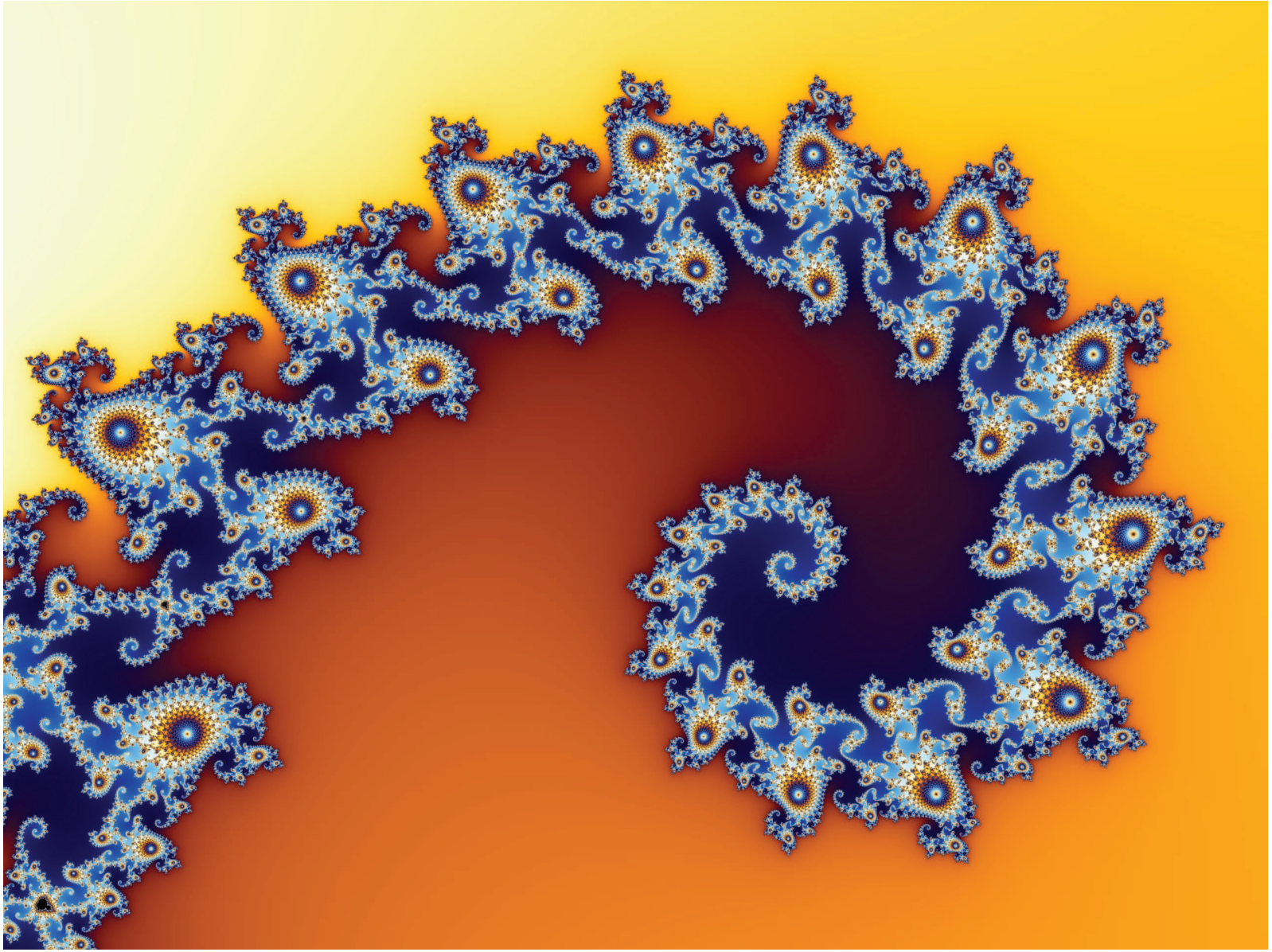

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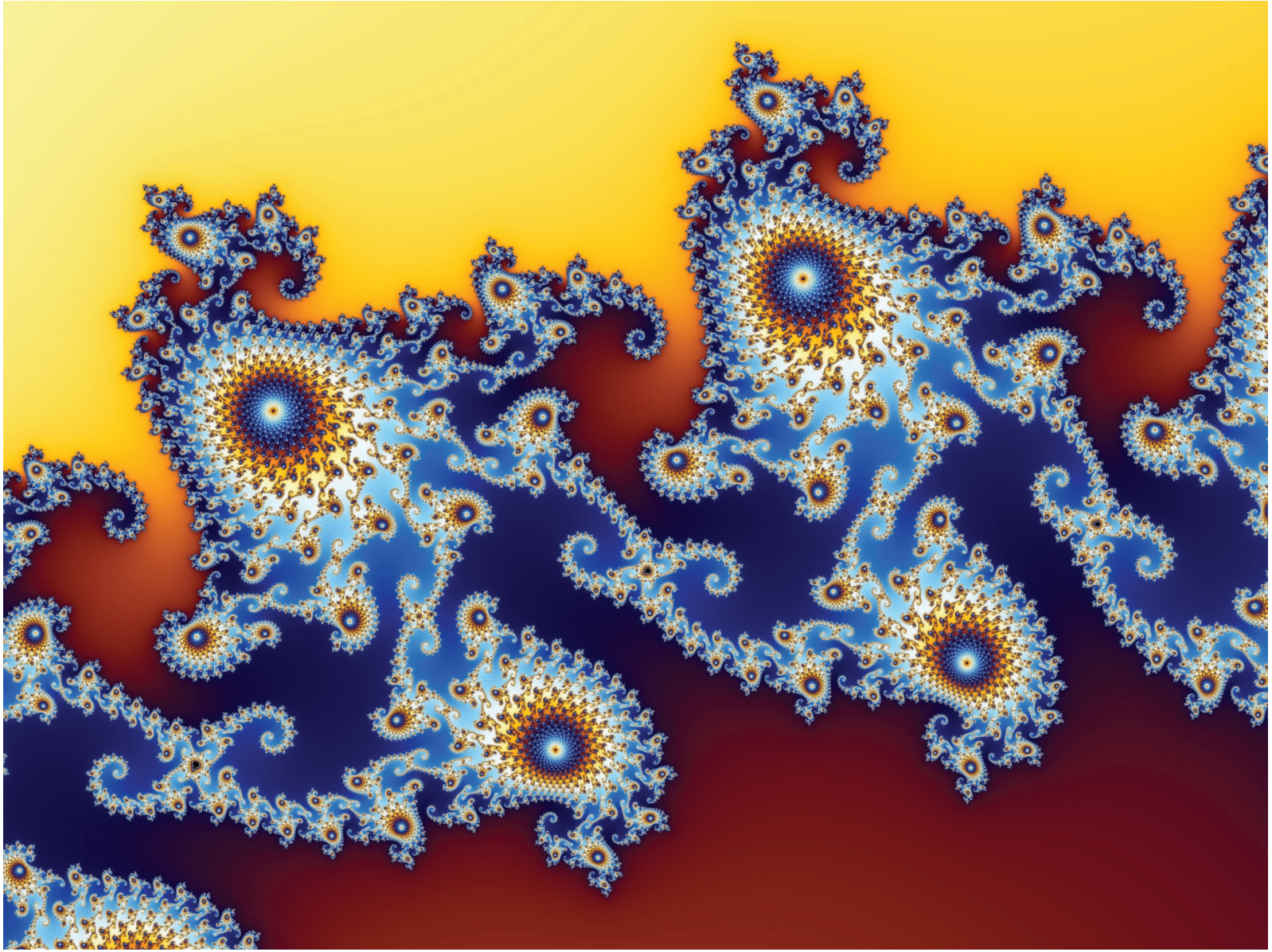

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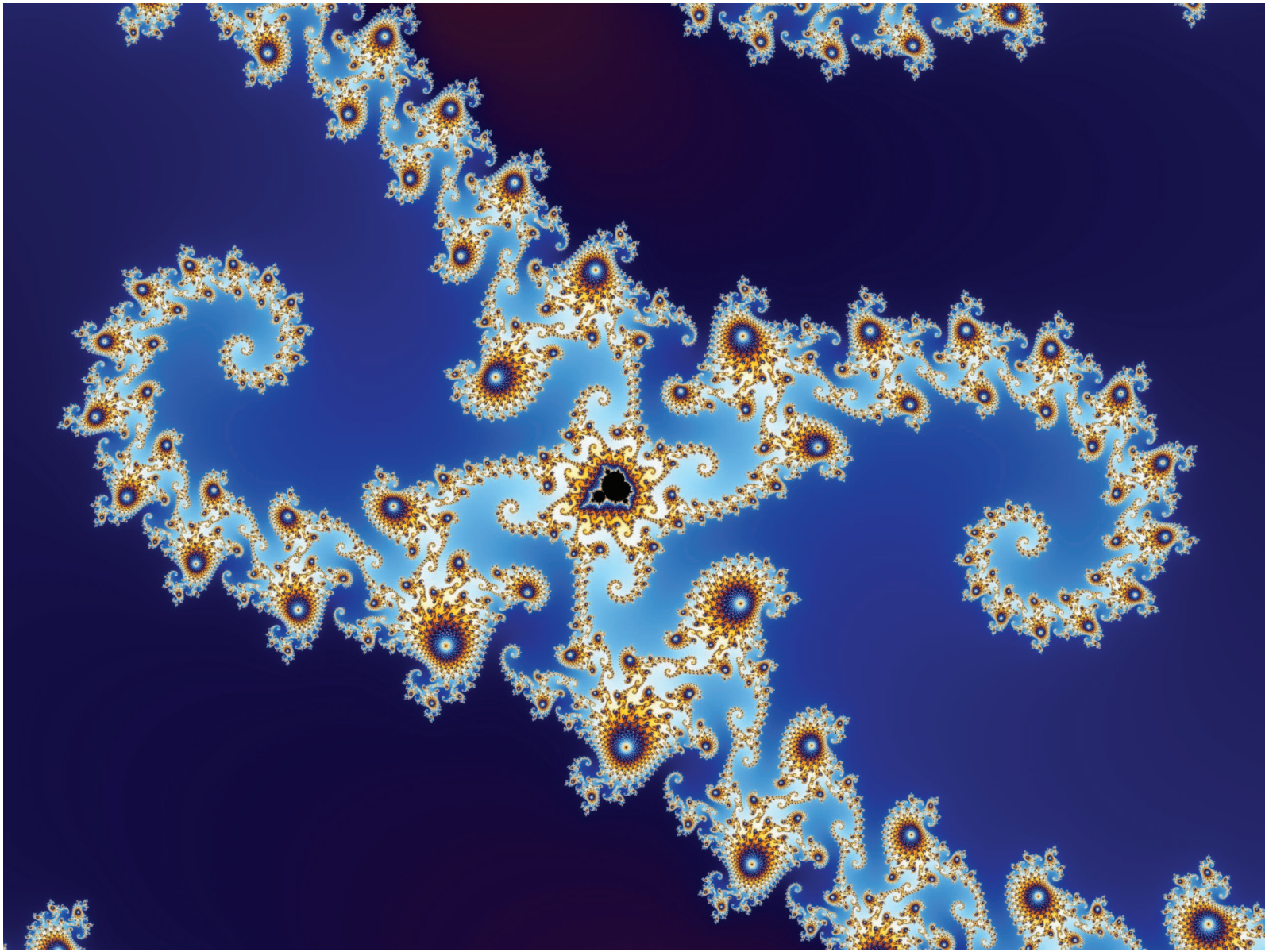

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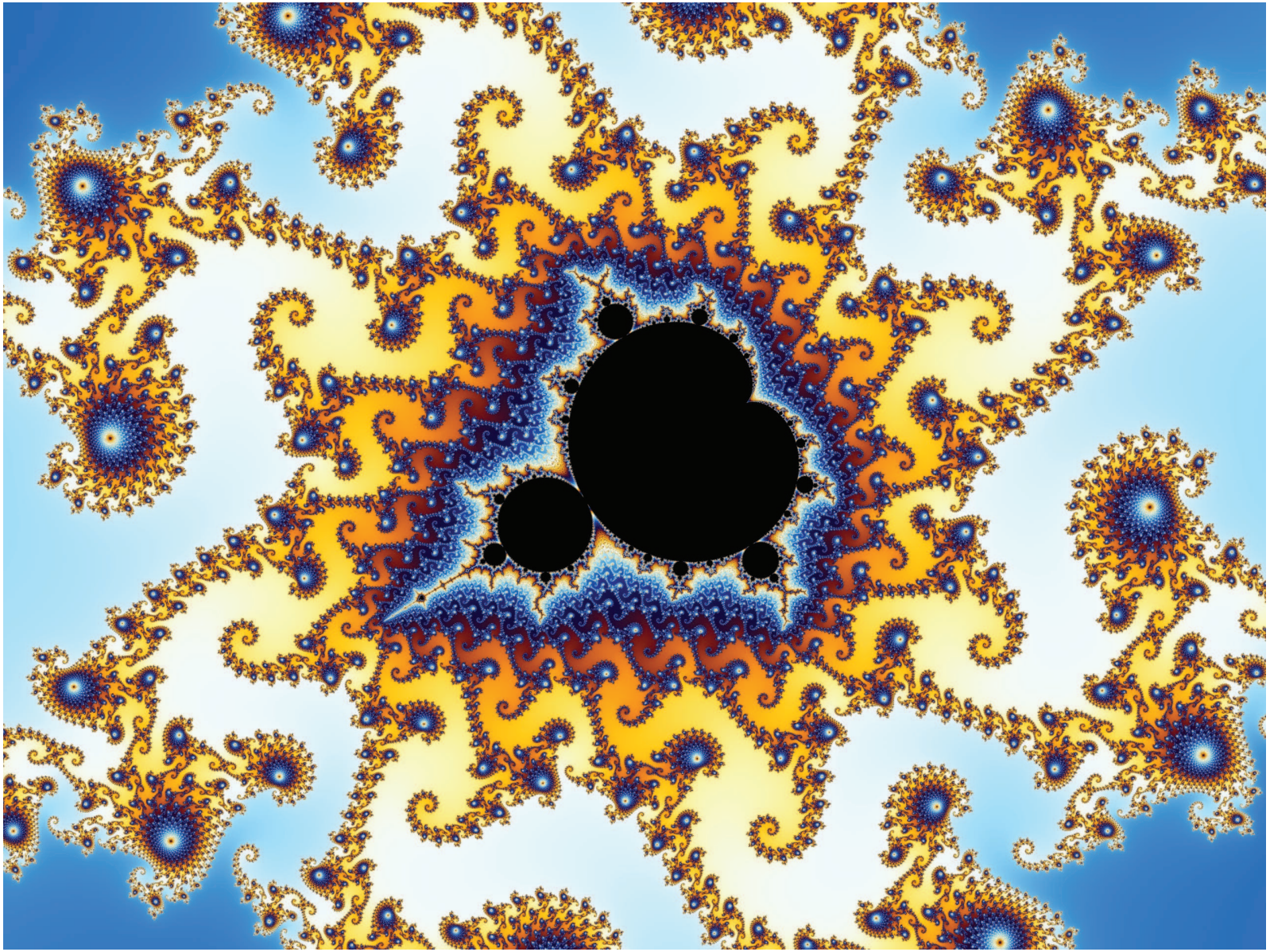

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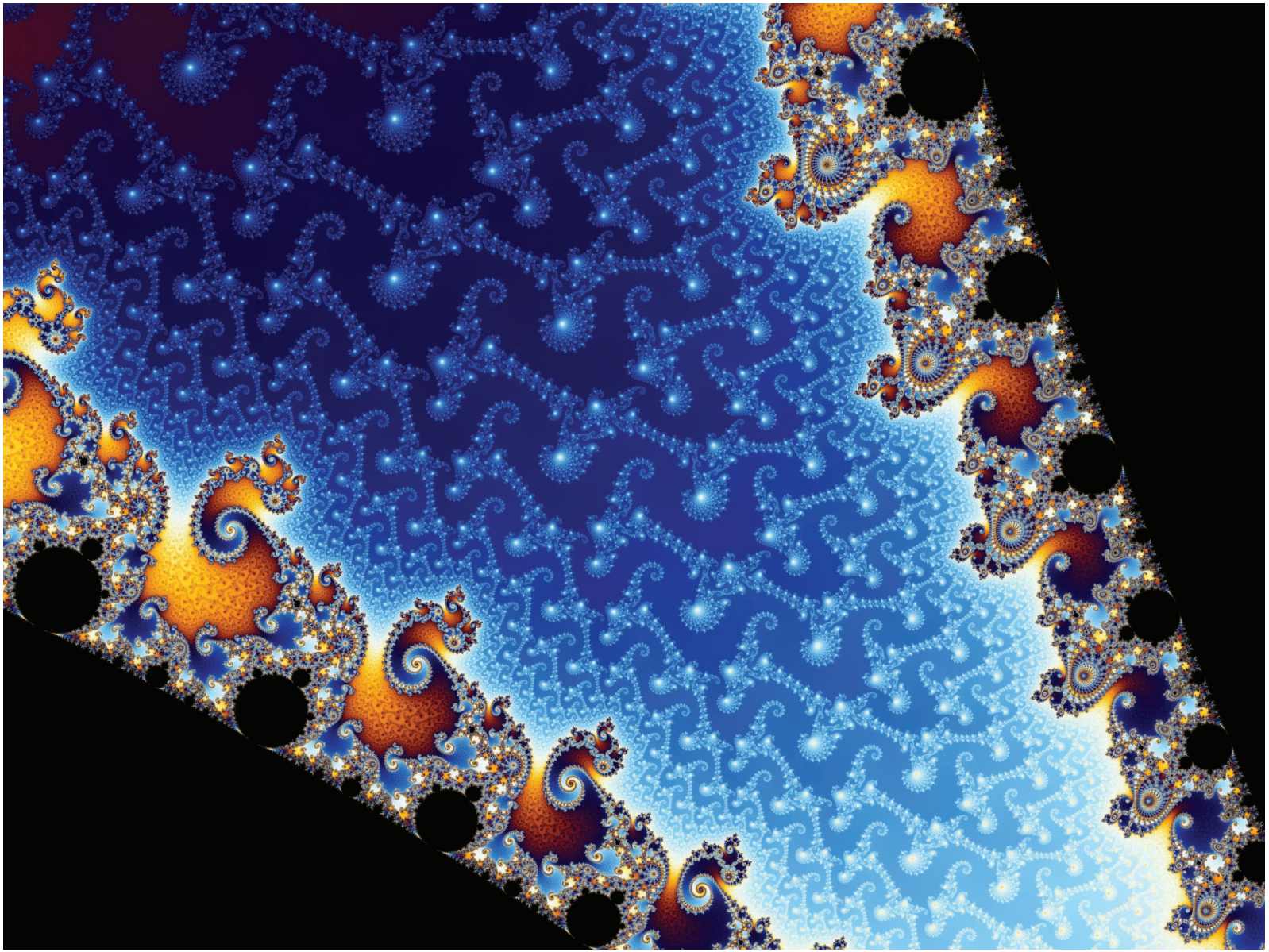

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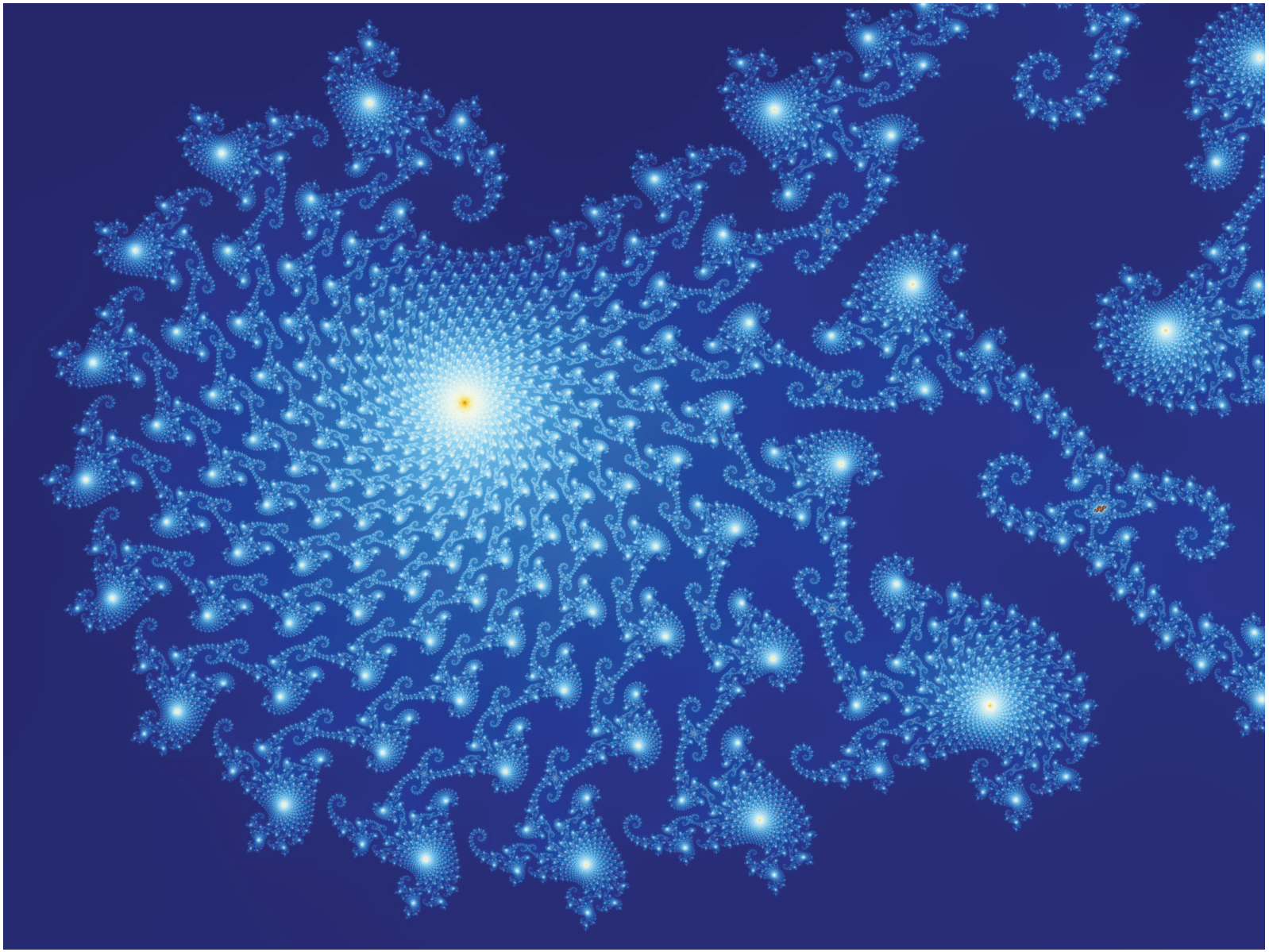
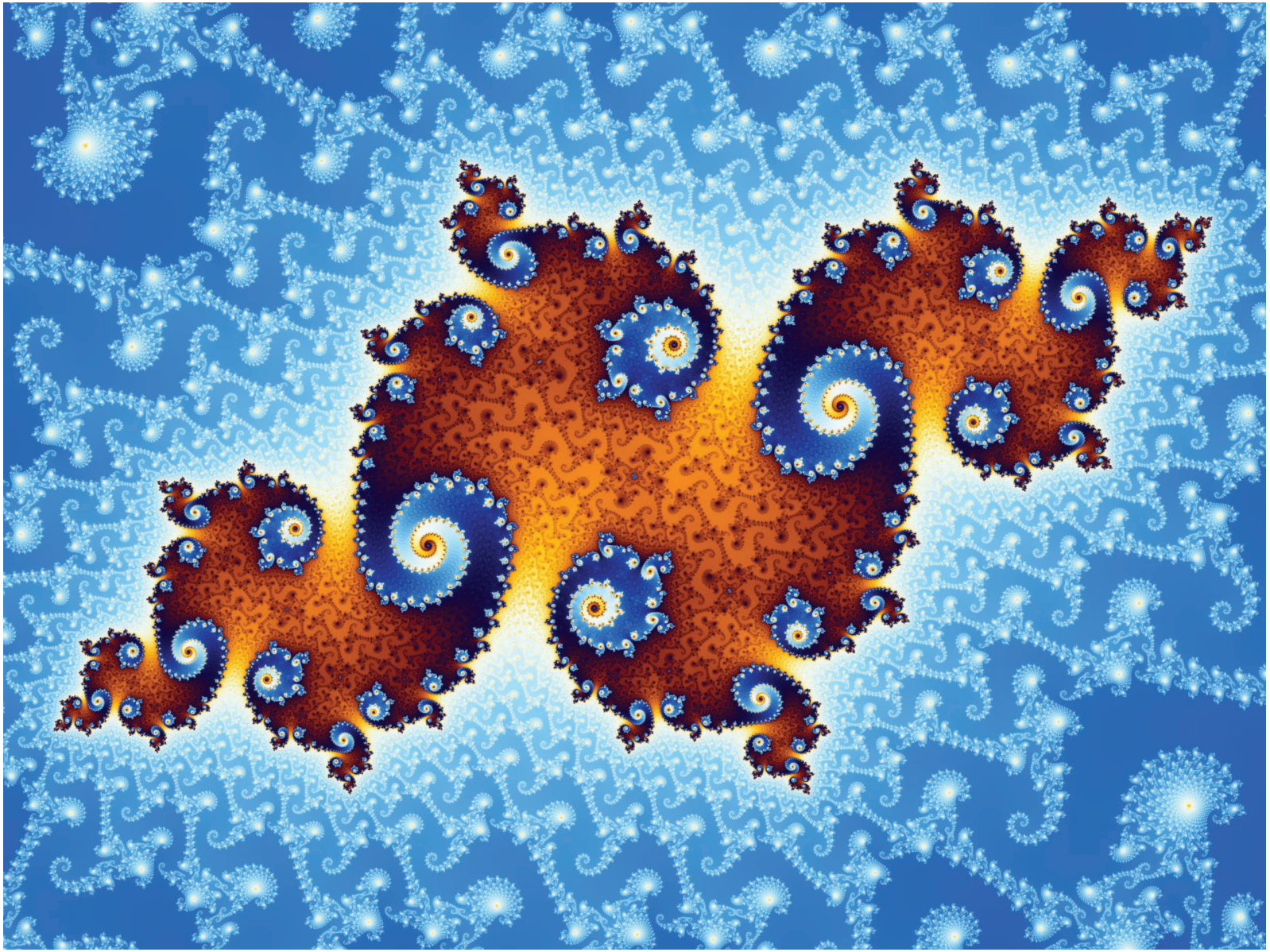
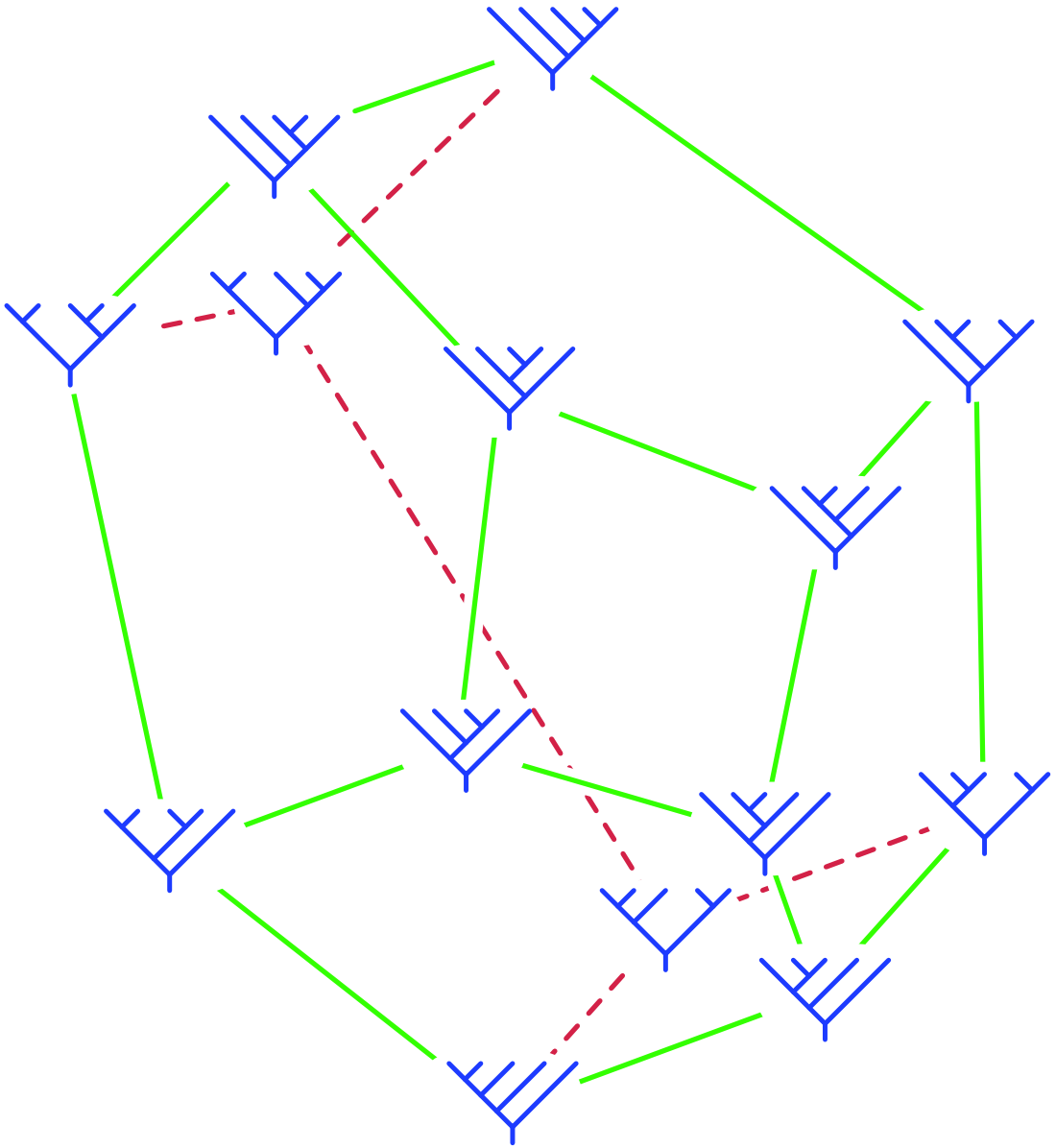


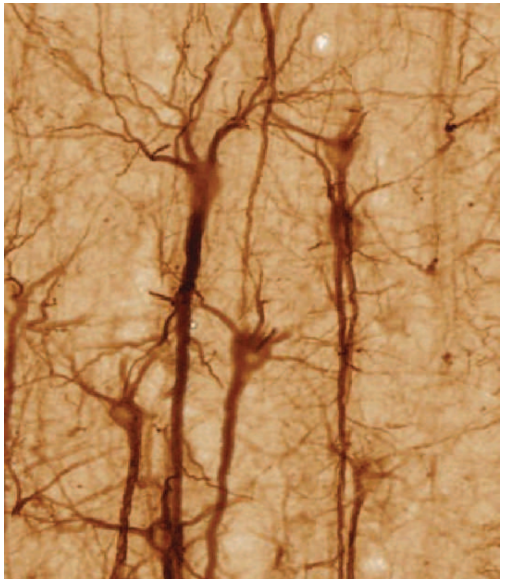
Mandelbrot set.



Back to binary trees.



Trees



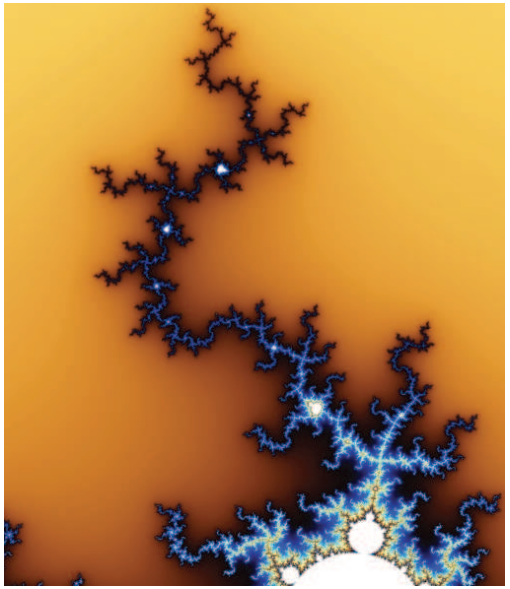


Image credits: Wikimedia.

Trees

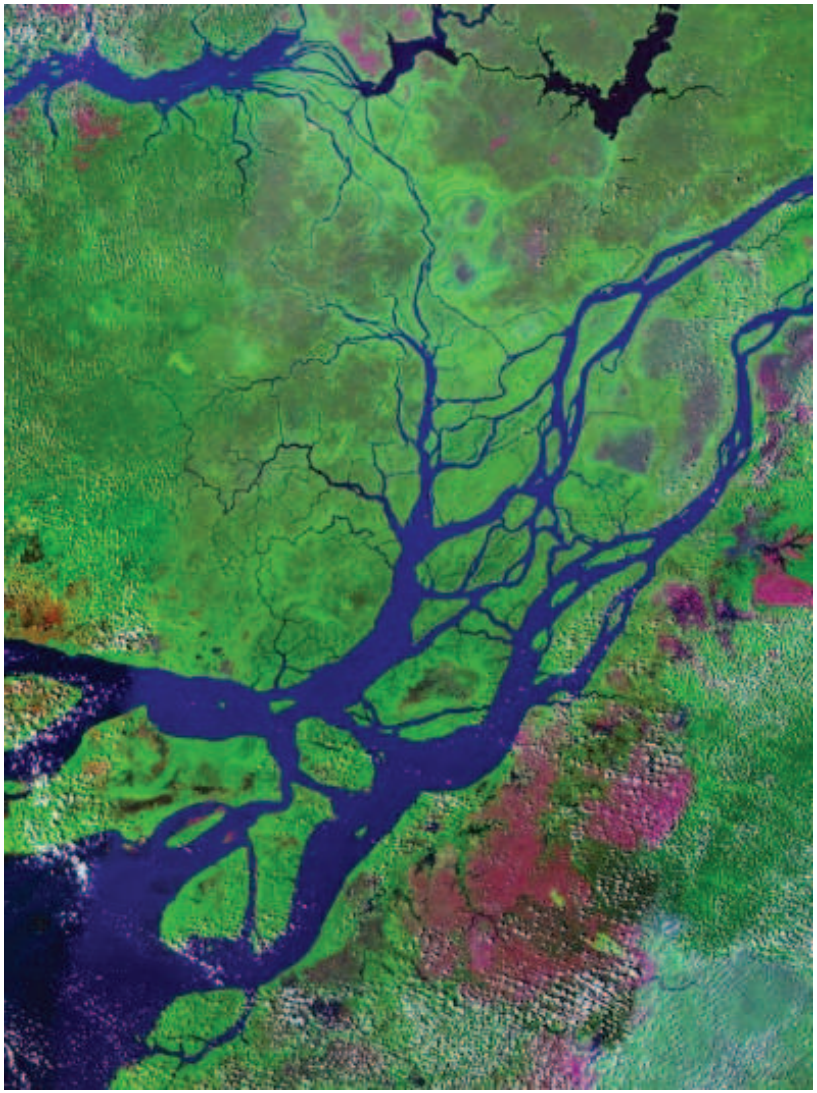


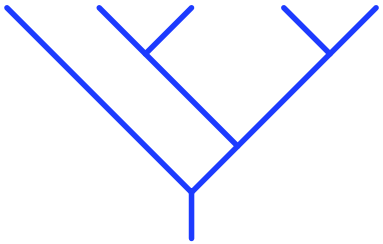
Image credits: Google Earth, Wikimedia.

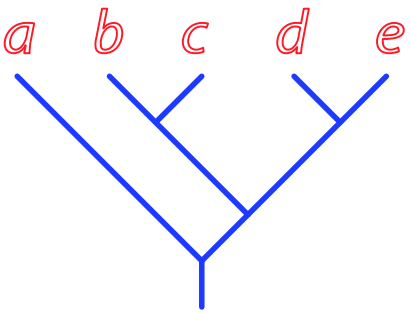
Trees



Image credits: Google Earth, Wikimedia (you may guess which!).

Labeled tree.





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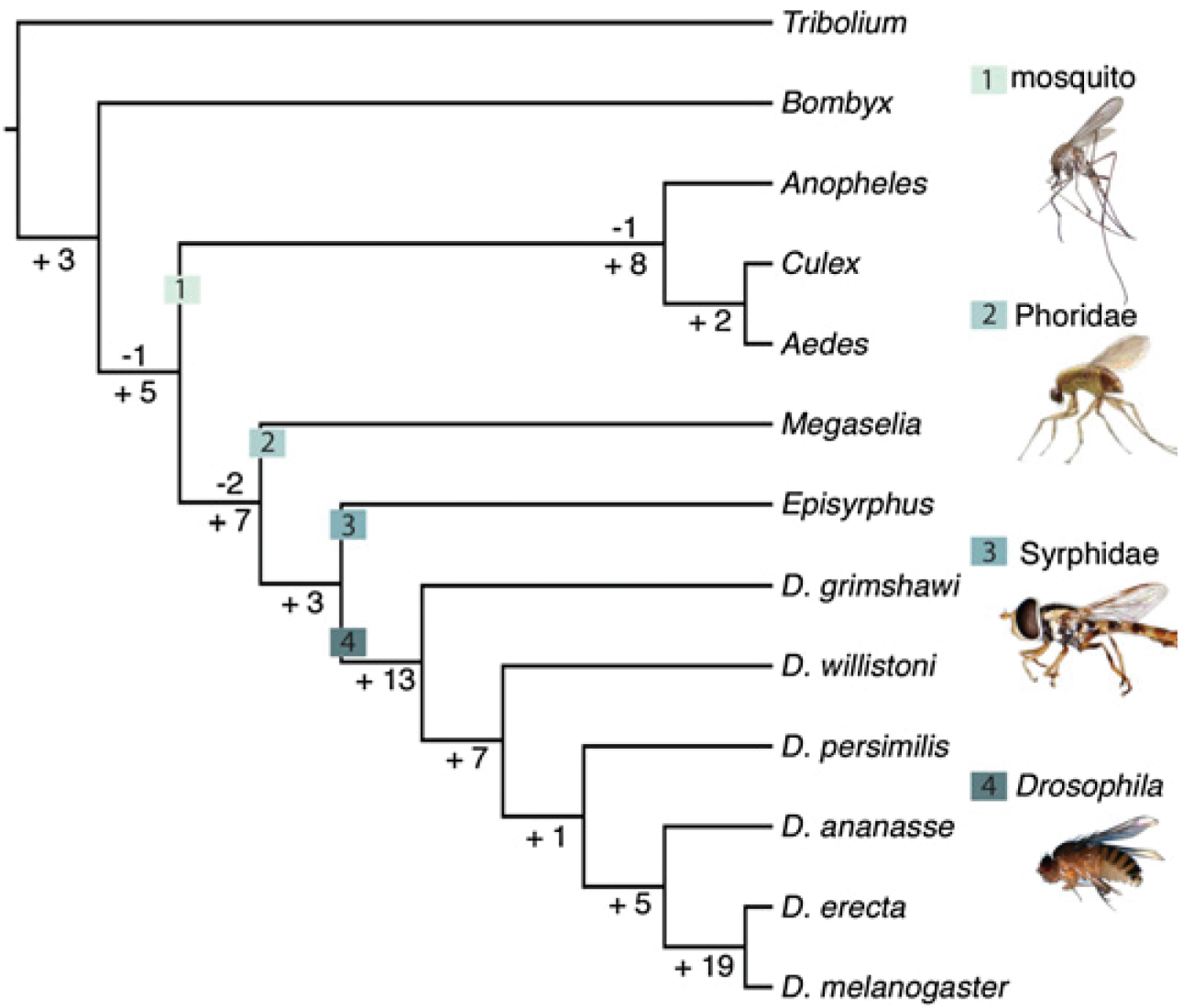
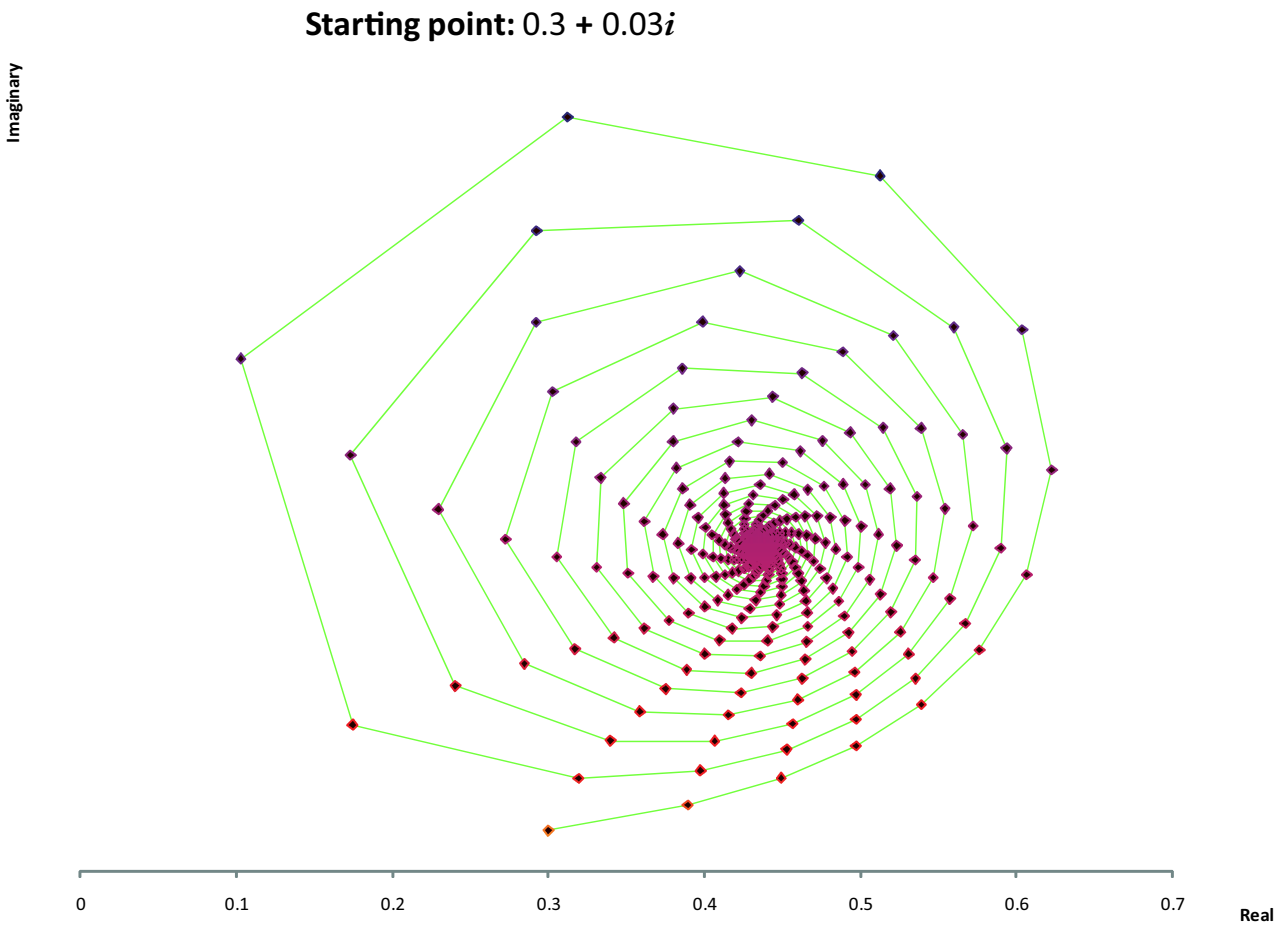


Episodic radiations in the fly tree of life, Wiegmann et.al. PNAS 2011

Thanks!



There are many more patterns to be discovered than there are already known...